Systematic Information Manipulation by Financial Intermediary

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Abstract

We study real effects of intermediation in a dual financial system that features coexistence of an open exchange and OTC bilateral market. Both markets trade futures on commodity, but volumes traded on bilateral market are hidden. That friction obstructs learning, as in order to correctly predict future spot price, uninformed traders have to infer realized productivity shock only from a fraction of total hedging demand. Financial intermediary may strategically shape producers choice of trading venue and impose a non-revealing equilibrium that features i) non-revelation of realization of aggregate productivity shock; ii) feedback effect from financial sector to real sector; iii) inefficient production decisions; iv) higher spot price volatility. We study welfare implications of financial intermediation.

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1 Introduction

Major financial intermediaries, such as Credit Suisse and Morgan Stanley, often perform multiple activities, at the same time lending to companies and serving as large security dealers. The provision of risk management services to commodity producers seems to be a logical extension of standard lending practices. Not surprisingly, as hedging is often considered as a precondition for getting funding, large financial intermediaries also help with arranging hedging transactions. The producers are advised what fraction of production to hedge, how long the hedging horizon should be, and more importantly what instruments to use. The choice of instruments involves a crucial choice of the trading venue, as the same contracts are traded on the open exchange and on the over-the-counter market. But do financial intermediaries have incentives to strategically manipulate the choice of exchange? Can financial intermediation lower welfare?

Our first observation is that while same futures contracts are traded at both venues, they differ substantially along an important dimension – transparency. Only volumes traded on the open exchange are publicly observable. But knowledge of total hedging demand and thus total production is needed to forecast the spot price. Thus if open exchange volumes are not fully revealing, in other words if the market does not know what fraction of total demand is traded on the open exchange, the equilibrium may not be efficient. However, non-revelation may be desirable for a sophisticated monopolistic intermediary, who is able to shape informed agents choice of the market and thus impose non-revealing equilibrium. The paper studies coexistence of an open exchange and a bilateral market, and shows that a strategic intermediary prefers an equilibrium that features i) non-revealing of realization of aggregate productivity shock; ii) feedback effect from financial sector to real sector; iii) inefficient production decisions; iv) higher systematic volatility.

We consider a two period model of production, hedging and consumption of a commodity. In the first period financial market opens and production of commodity is undertaken, in the second period commodity is traded on the spot market and consumed. Spot price is endogenous, it depends on the amount of commodity produced and demanded. There are two aggregate shocks in the economy: productivity shock and demand shock. To simplify we assume only two productivity states - high and low. Demand shock is realized in the second period and represents exogenous and unforeseeable shifts in the demand for commodity. Thus, there is a scope for risk sharing. Financial market exists to transfer risk from risk averse producers to competitive risk neutral traders each investing one unit of consumption good. Financial market features coexistence of the open exchange and a bilateral market. On the open exchange both parties are required to put up an initial amount of cash, the margin. Margin or collateral represents a particular fraction of the notional amount, the interest rate is the same for producers and traders. On the bilateral market, a rate at which risk neutral trader may attract financing for a specific trade and thus the futures price agreed upon, depends on the producer's type. Local factors may matter, as some producers may have very good location next to the main pipelines or to the main officially designated delivery points as Cushing, OK, or may have a good credit history. Thus efficiency of trader-producer match on the bilateral market matters. Producers endogenously self-select to trade either on open exchange or on the bilateral market based on their ability to create an efficient match on the bilateral market. We argue that monopolistic intermediary is able to rehypothecate and thus more efficiently use the collateral, and the intermediary also may secure the trade. Therefore, on behalf of over-the-counter traders the intermediary offers new contracts to a chosen fraction of producers, that would otherwise trade on the open exchange. A contract assumes the market price observed on the open exchange and requires a producer to put exactly the same collateral. The producers are indifferent, and accept new contracts, thus moving their hedging volume away from the open exchange. The intermediary keeps the return on rehypothecated collateral.

In the absence of productivity risk or in the presence of perfect information, the system works perfectly fine and only demand risk is shared. However, productivity shocks exist as well, and it is not possible any longer to hedge the demand risk separately from productivity risk. The problem is that producers privately observe the realization of aggregate productivity shock, hence, all producers may assess total production volume of commodity and thus the distribution of the spot price. Hence, producers have informational advantage relative to risk neutral traders who have to infer total production from observable traded volumes on the open exchange and the futures price. Of course, if the futures price (or volumes) is revealing, information advantage disappears, and every agent
correctly updates his beliefs about the future spot price and accounts for the total production volume. However, in non-revealing equilibrium, producers are able to forecast pricing error done by the market. That in turn affects their production decision, and hence, spot price distribution, and, finally, systematic volatility. Thus there is a feedback effect from financial market to real sector. And information friction is at the core of this effect.

Absence of full revelation means that allocation of producers among the markets in high and low productivity states is such that observable market statistics, such as open exchange traded volumes and open exchange futures prices are non-revealing, they take the same values in both states. Hence, productivity shock cannot be inferred by uninformed traders and prior probabilities are used. Why would the intermediary prefer non-revealing equilibrium? We abstract from proprietary trading, the intermediary only connects producer and trader and keeps the return on rehypothecated collateral. The intermediary gets larger profit, if more producers prefer open exchange, and thus may accept the contract. But the intermediary also accounts for information externality. The larger is the mistake made by the uninformed traders on the open exchange in the direction of underestimating the production and thus offering higher price, the more attractive becomes alternative of going to the open exchange and hence even more producers go to the open exchange, and hence even larger number of intermediated contracts can monopolistic intermediary offer. In equilibrium the intermediary chooses optimal number of contracts to offer to producer given market beliefs structure and endogenous subdivision of producers.

We show that any pooling or non-revealing equilibrium is characterized by higher volatility of production and higher spot price volatility. As for welfare, consumers and producers that trade on the open exchange get higher utility in the pooling equilibrium at the expense of traders of the open exchange. Each trader is able to trade only one unit of consumption good, thus conditional on ability to trade he correctly expects zero profit using prior probabilities. The problem is that ex ante, before joining the market, the trader has to account for the fact that production will be much higher in a state in which he would suffer losses, thus the probability to be a part of “unlucky” group is actually higher than the probability of being in a group that gets positive profit. Thus ex ante expected profit of all traders of the open exchange is negative, although each one of them using prior probabilities conditional on being able to trade gets zero return on his unit investment. The problem is not accounting for execution risk. Finally, the intermediary prefers one equilibrium or another depending on the parameters. Methodologically, we offer a setup that contains a financial friction and offers non-revealing equilibrium without noise traders assumption, thus allowing a more rigid welfare analysis.

The papers closest to ours include Lizerri (1990) and Duffie and Strulovici (2012). Lizerri (1990) studies the optimal choice of disclosure rule by monopolistic certification intermediary in a seller-buyer setup, with informed seller and uninformed buyer, under existence of a free perfect quality test whose results can be made public in an arbitrary way by the intermediary. The intermediary strategically manipulates information in order to collect higher fees. Our paper differs from information intermediation literature (...) in one very important way: the intermediary in our paper only indirectly affects transparency of the market, shaping the fraction of hedging demand that shows up on the open exchange, we also do not assume any proprietary trading. Duffie and Strulovici (2012) model the behavior of intermediaries that endogenously chose effort to re-balance the distribution of capital across exogenously segmented markets. The intermediary searches for investors in the overcapitalized market with low return and offers them to move their capital to undercapitalized market to earn a higher return, for that he charges a fee that is proportional to the gain from the move. The greater the heterogeneity in capital levels across the markets, the more intensive are intermediaries’ efforts. In a simple case the behavior of intermediaries demonstrates S-S pattern, with inaction region, effort switches from minimal to maximal levels. The feedback effect is similar to Sockin and Xiong (2013). Our paper compliments also financialization and speculation literature that tries to explain the higher volatility of oil prices in 2008 and relate to the institutional change in financial sector. Recently JPM testified before CFTC commission saying that i) “investors aid in efficient and smooth price discovery”; ii) “JPM is aware of no credible academic study or analysis that demonstrates that the presence of non-commercial interests in commodity markets has been detrimental and many that reach the reverse conclusion.”. That paper is an attempt to be a piece of such academic study.
2 Empirical evidence and literature

2.1 Role of G14 intermediaries in the OTC market

We study the behavior of major financial intermediaries, that are usually referred as G14 - group of 14/15 major derivatives dealers. Those include: Barclays Capital, BNP Paribas, BOA-ML, Citi, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JP Morgan, Morgan Stanley, Nomura(joined in August 2011), Royal Bank of Scotland, Societe Generale, UBS, Wells Fargo.

Financial intermediaries offer risk management services  Anecdotal facts show that companies often write hedging and lending deals with the same banks

- Energy companies need constant access to credit to either start developing new project or continue doing their business. A crucial requirement for getting funding is often a revenue hedge requirement.
- Many producers prefer an integrated set of services and are likely not to be experts in financial markets

According to IHS (2013) report: “the provision of risk management services to commodity market customers is a logical extension of this traditional lending practice. Banks are able to provide unique risk management services to diverse commodity market participants due to their credit capabilities and commodities market expertise.”. Hence, financial intermediaries may provide access to capital and risk management services as an integrated package.

G14-intermediaries stand between producer and consumers wishing to hedge price risk and traders wishing to provide insurance. The coexistence of two exchanges is a crucial prerequisite for such kind of business activity. Intermediaries stand as an opposite party in hedging deals, accumulate aggregate net position and then offload it to the open exchange to get a balanced position if needed. However, during that process there are plenty of degrees of freedom: financial intermediaries may influence and shape hedging decisions of producers, including instruments and hence markets used, may decide on how much risk to offload immediately or later, the extent of aggregation before offloading the residual position to the market. Examples of different decisions are multiple:

- Contracts sent to the open exchange: Mexico’s oil hedging program, 2008-present day.

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1"The working capital requirements for independent refineries are horrendous,” says Turner, who now heads Turner, Mason & Company, a consultancy in Dallas. “You need to have a letter of credit for this, and a letter of credit for that, and if you need shipping of foreign barrels, the requirements for working capital are just enormous. And few companies can really afford that.”


2"Finally, as a condition precedent to the funding of the EPOP Revolving Credit Facility, EPOP is required to hedge 75% of proved developed producing production through 2010”. EXCO resources, 10-K form

3J.P. Morgan provides clients with hedging services that result in commodity price certainty and capital relief.” - JPM brochure

4"Staying on top of the market takes the best information and the brightest people. J.P. Morgan is proud of the work produced by our independent research team and the insightful market commentary provided to our clients. Our research reports help both corporate hedgers and institutional investors navigate the risks and opportunities of commodities markets globally.”

5The vast majority of large American companies, and a significant percentage of all other American companies, enter into OTC derivatives transactions to hedge risks that arise out of their core businesses. Financial intermediaries such as J.P. Morgan stand between those entities that want to offload risk, typically in the OTC market, and those entities that want to take on risks. Intermediaries aggregate risks assumed in this way from multiple parties and manage the resulting net positions on a portfolio basis. Typically, dealers flatten out such net portfolio risk positions on designated contract markets and exempt commercial markets, which for ease of reference I shall refer to as exchanges. Thus, for risk management purposes, exchange and OTC markets are used interchangeably and the ability of a financial intermediary, or swap dealer, to use exemptions to position limits to effectuate OTC transactions is key to being able to provide risk management tools to its customers.” testimony of JP Morgan, CFTC, Hearing on Energy Position Limits and Hedge Exemptions, July 29, 2009.
Deals are arranged by Barclays, Deutsche Bank, Goldman Sachs and Morgan Stanley. Usually the country buys put options from the banks. To hedge their own exposure, the banks sell Nymex oil futures in the regulated market along the curve. To great extent hedging demand is send to the open exchange.

- Offered contracts, OTC:
  Shale gas companies hedging their exposure with OTC swaps.
  ‘BNP Paribas’ clients in the refining industry are becoming increasingly well versed in strategies that use put options, zero-cost collars or swaptions, which are options to buy swaps, such strategies are becoming more popular with refiners, while banks have become more likely to offer them to clients in the refining industry.  

Depending on the situation financial intermediaries may offer one or another contract and either translate it to the market or keep it on its balance and match it with the opposite deal with someone else, without going to the open exchange. However, even indirectly, the level of transparency on the financial markets might depend on the strategic decisions of G14 intermediaries. Because they may offer higher or lower degree of contract standardization. For example, to encourage the production of shale gas in US Volumetric Production Payments (VPP) contracts were offered, this enabled Chesapeake Energy and other companies to collect cash up front for the delivery of gas to customers in the future. Usual maturity of hedging contracts 1-2 years, whereas VPP were signed for 5-10 years. Similarly, the basis contracts were highly popular, that fix the price of gas in a local place wrt the price in Henry Hub. Hence, offering companies more or less flexibility to hedge their risks, the intermediaries might be able to affect the size of the markets even indirectly.

**Obscure nature of OTC market and physical market**  
Scarce public data on OTC market or the physical markets- is exactly what puts dealers to advantageous position. Whereas the market is not able to track all the deals, the dealers are in the perfect position to gain the relevant information.

- Physical market of natural gas is obscure. According to FERC 2009 survey, index publishers are deriving their index prices from a relatively small amount of gas volumes. For every MMBTu reported to an index publisher, more than 6 MMBtu rely on that price.

- Offtake deals with refiners give banks access to superior information and access to physical market of petroleum and oil. “Banks are tight-lipped about how exactly they benefit from supply and offtake deals with refiners. It is clear, however, that such agreements offer the banks new physical oil trading opportunities, increase their knowledge of oil and product flows, and give them access to refiners’ storage capacity”. “In 2010, Goldman Sachs entered a two-year supply and offtake deal with Alon Refining Krotz Springs, which runs a refinery in Louisiana. Under the terms of that deal, Goldman supplies 100% of the refinery’s crude and purchases all its refined product output. All sales are made at market-related index prices Morgan Stanley, JP Morgan and Barclays Capital have all done similar deals, although the details of each one vary.”

- G14 made deals with counties and utilities, gaining total coverage of informational space

- In an eight-week Bloomberg News survey conducted this year, commodities traders expressed skepticism about benchmark prices, with 85 traders and analysts, of 270 surveyed, saying they have little confidence in the assessed prices of crude, metals and iron ore.

So to sum up G-intermediaries consolidate information regarding hedging demand and future and current supply of commodities. They may send their customers to the market of their own choice depending on the strategic

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[8] LA county metropolitan Authority (deal with Barclays, RBC Capital, Citibank 90-95% hedge).

State Water Resources deal with Morgan Stanley. “During fiscal 2006, the System was a party to certain power purchase contracts with Morgan Stanley. These contracts require the System to pay market rates for the natural gas used to produce such power. As a result, the System established a natural gas hedging program designed to stabilize the cost of its natural gas. This hedging program causes the System to enter into contracts for the future purchase of natural gas at known costs to hedge against market swings in price.”

Same for other years.
considerations. Obscure nature of OTC market and physical market provides perfect environment for strategic information manipulation. We will describe current situation with the data on the size of OTC market in the next chapter.

**Manipulation of information and trend towards opaqueness**  
Manipulation of information by major financial intermediaries is longer considered as impossible. In the last few years a number of scandals occurred regarding the role of G14 intermediaries in different instances of price manipulations, including contango in the oil market, LIBOR manipulations, insider trading and front runs. In the most recent scandal financial intermediaries have been accused of the manipulation of Brent prices. The ability of major financial intermediaries to participate in the physical market and affect market outcomes is now under investigation in a Senate hearing. Hence, neither manipulation should not be particularly shocking given these empirical facts of the financial and physical markets. Duffie(2008) points out that “the profits of a dealer depend on the volume of trade it handles and on the average difference between bid and ask prices, which in turn depends on the degree to which the dealer’s customers are likely to have information relevant to prices available elsewhere in the market. Dealers therefore prefer at least some degree of market opaqueness. as pre-trade and post-trade price transparency increases, dealers have incentives to narrow their bid-offer spreads in order to compete for trading opportunities. If price transparency is too great, however, some dealers may lose the incentive to intermediate, given their fixed costs and the risk of adverse selection by informed customers.”

But before going to the theoretical model we would like to assess the size of the each market: fraction of producers that deal with intermediaries. to be completed

### 2.2 Efficient use of collateral by intermediaries

to be completed

Singh (2009): “Based on discussions with collateral teams at large banks (since hedge funds do not provide this information), about USD 1 trillion of the market value of securities of the hedge fund industry was rehypothecated, as of end-2007.”

Greg O’Sullivan. “Total collateral in force across the OTC derivatives market is currently estimated at $2.9 trillion, of which about 80 percent is held in cash.”

### 2.3 Literature

to be completed

- Segmented markets / “institutional finance”
- endogenous degree of intermediation, intermediation gain depends on degree of heterogeneity in capital levels: Duffie and Strulovici (2012)
  - harmful arbitrage allocates risk to those who least understand it: Weyl (2007)
  - slow capital, fast information: Goldstein, Li, and Yang. (2013)
- Dual financial system

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9 The traders allege that supermajors Royal Dutch Shell, BP and Statoil, along with trading houses Morgan Stanley, Vitol, Trafigure Beheer, Trafigure and Phibro Trading joined forces to manipulate Brent crude oil prices and Brent futures contracts traded on NYMEX between February 2011 and September 2012... Platts’ methodology can be easily gamed by market participants that make false, inaccurate or misleading trades... “It’s a very obscure market,” David Kovel, a lawyer for the traders, said of oil traded outside of exchanges such as the Nymex. “To outsiders, it can seem impenetrable. Specialists and specialty traders in the market can take advantage of this obscurity.”


10 What we are seeing as more and more cases come out is a systematic pattern of unethical and often illegal behavior in the pursuit of profit at clients’ expense.” F. Allen
- opaque OTC market and a transparent exchange: Bolton, Santos and Scheinkman (2013)
- different degree of anonymity: Grammig et al. (2001)

• Strategic information manipulation:
  - information intermediary: Lizzeri (1999); Farhi and Tirole (2013)
  - advisors manipulating the expectations of naive investors: Hong, Scheinkman, and Xiong (2008)
  - fear of opportunism by informal financiers: Kondo and Papanikolaou (2011)

• Security design/financial innovation
  - Duffie and Jackson (1989), Simsek (2013)


• Intermediation: Babus and Hu (2015)

3 Baseline Model

We consider a two period model and two goods: commodity and consumption good. In the first period futures market opens, in the second period - spot market. There are two shocks in the system, productivity shock is realized in the first period and is observed privately by producers. Demand shock is realized and observed by everyone in the second period. Basically futures market exists to hedge the demand shock, however the productivity shock affects market’s ability to do that.

The economy is populated by four types of agents in the economy, consumers that consume goods bought in the spot market, producers that produce and hedge part of their production on the futures market, traders that provide hedging services and a monopolistic intermediary that affects the way futures market operates.

3.1 Consumers

Preferences are given by $U_{\text{consumer}} = C$, where $C$ denotes amount of consumption good. Consumers buy commodity on the spot market and produce consumption good out of it and consume it, but their production technology gets hit by a demand shock. Facing the realization of the demand shock $\xi$, consumers maximize

$$U_{\text{consumer}} = C$$
$$C = \xi \frac{X^{1-\rho}}{1-\rho} - PX$$

Hence,

$$\xi X^{-\rho} - P = 0$$

$\rho$ represents elasticity of demand, high $\rho$ means inelastic demand.
3.2 Producers

Producers produce commodity, let $X$ denote the total production volume. Then the cost of production in terms of the consumption good is $\frac{\gamma X^2}{2}$, where $\gamma$ denotes productivity shock that is privately observed by the producer. Productivity takes two values: $\gamma_H$ with probability $\pi$ and $\gamma_L$ with probability $1-\pi$. Producers also choose what fraction of their production volume to hedge, we denote that fraction as $\nu$. We do not allow speculation, producers can hedge only as much as they produce and cannot buy futures, therefore $\nu \in [0,1]$. Hedging requires margin - a fraction of the notional amount of the deal in terms of consumption good should be delivered, we denote that fraction as $\bar{c}$. That means that producers lose interest that could earn at that amount with the rate $\tilde{r}$, hence total opportunity cost equals $\bar{c}\tilde{r}\nu FX$. Preferences represent risk aversion of producers and are given by $\tilde{U}_{producer} = E\xi C - \frac{\psi}{2} Var\xi C$, degree of risk aversion is parametrized by $\psi$. Producers anticipate futures price $F$ and maximize utility with respect to total production $X$ and amount of hedging $\nu$.

$$\tilde{U}_{producer}(X,F) = F(1-c_{fee})\nu X + EP(1-\nu)X - \frac{\gamma X^2}{2} - \frac{\psi}{2} VarP(1-\nu)^2 X^2$$

First term denotes revenue from hedged fraction $\nu$ of total production $X$, second term represents revenue from unhedged fraction of production, third denotes production costs, and last part represents risk aversion, with risk coming from unhedged part of production volume vulnerable to demand shock. We assume that $\psi$ is big enough so that $\nu > 0$.

$$X = \frac{F(1-c_{fee})}{\gamma}$$

and

$$1-\nu = max\left\{\frac{EP-F(1-c_{fee})}{\psi VarP X}, 0\right\}$$

If futures price is too high relative to expected spot price based on information that producers have, than they would prefer to speculate, sell more gas than they actually produce, $\nu > 1$, but we preclude them from doing that.

Production depends on the futures price $F$ observed on the open exchange. However, when it comes to actually write a hedging contract, when production is already determined, a producer gets to observe available for him price at the bilateral market, $F_{B,i}$, it will be clear later how it is determined. SO producer may either buy contracts on the open exchange under price $F$ or he may also buy futures on the bilateral market with $F_{B,i}$. Producers are free to choose a market with the highest price, however production is fixed at $X^*$ and cannot be altered. Therefore, final utility equals the previous utility plus the additional term which is non zero and positive only if bilateral price is higher and hence indicator $I(F_{B,i} > F) = 1$:

$$U_{producer} = \tilde{U}_{producer}(X^*,F) + I(F_{B,i} > F)(F_{B,i} - F)(1-c_{fee})X^*$$

Hence, we will have endogenous demand for open exchange futures: $\mu$ - a fraction of producers prefer to hedge using open exchange futures.

3.3 Financial Market

In this section we describe the structure of the futures market and the role that is played by monopolistic intermediary. We consider coexistence of the open exchange with OTC market: i) open exchange market reveals both
cumulative trading volume and price; ii) all of the deals on the OTC are private, therefore trading volumes and prices are not revealed. It is important to notice, that spot price depends not only on demand shock, but on productivity shock as well, through production volume. Therefore, in order to correctly price futures contracts, traders would have to make inference regarding total production volume. Total production volume or total hedging demand are not observable. Inference must be made based on the open exchange traded volumes and structure of the economy. And open exchange volume does not have to be fully revealing, that would depend on the behavior of monopolistic intermediary. So what is the role played by monopolistic intermediary? The power of monopolistic intermediary to affect the allocation of traders between exchanges stems on their ability to use the collateral more efficiently and on their ability to perform “clearing house” services on OTC market.

When producer prefers to go the open exchange, or when observable open exchange price is higher than available to him bilateral price, he has to provide \( \bar{c}FX \) as collateral, thus effectively losing \( c_{fe}FX \) units of consumption good\(^{11}\). However, as empirical evidence suggests the intermediary is usually able to use the collateral more efficiently, for example rehypothecating it\(^{12}\). Therefore, the scope for mutual improvement arises: and the intermediary may perform as a “clearing house” or “exchange organizer”, connecting producers and sellers bilaterally offering both to make a deal on observable market price \( F \) (referencing) and keeping the return on collateral to himself. The producer still posts the same collateral as the one he would on the open exchange, but the intermediary is able to use it more efficiently, hence getting the return on it. The switch from open exchange contract to the intermediated one is irrelevant for producers and traders - prices and costs are the same. But that makes a huge difference on aggregate, as it affects inference abilities of the market as a whole.

So there is a tradeoff: on the one hand, the intermediary effectively “creates” new resources out of nowhere, and that should be beneficial to the overall economy - collateral money stop being a dead weight on the brokerage accounts. However, the system is vulnerable to strategic manipulation of information by the intermediary. He may prefer to improve in terms of the collateral usage only a particular fraction of open exchange deals such that the open exchange volumes would stop being revealing and that would create inference problem of the market traders. That is beneficial for intermediary as it places producers to informationally advantageous position relative to traders, hence there would be more of them who would prefer to trade on the open exchange, and hence the intermediary would have a bigger pie to divide. So the intermediary strategically chooses the fraction of the pie, knowing how it affects the size of the pie.

Table 1 sums up the structure of the financial system. The intermediary faces endogenous demand for open futures contracts, \( \mu \), and decides what fraction of that demand to “steal”, \( (1 - \lambda) \). Therefore, three markets coexist:

- open exchange with the size of \( \lambda \mu \)
- OTC market
  - bilateral part, \( (1 - \mu) \)
  - OTC “controlled” by the intermediary, \( (1 - \lambda)\mu \)

Now let’s look at the details of each market separately.

### 3.3.1 Open exchange

We consider competitive futures market with traders having access to capital market with interest rate \( \tilde{r} \). We assume that each trader has one unit of consumption good. He can invest it on the capital market or, alternatively, he can buy futures contracts, that would mean that he promises to take the delivery or to buy the commodity on a fixed predetermined price \( F \) in the future. Commodity can later be sold on the spot market at the price \( P \). Traders are risk neutral.

\(^{11}\)Although we do not microfound the existence of collateral, we have in mind that the need for collateral comes from any kind of default story\(^{11} \) for details).

\(^{12}\)Based on discussions with collateral teams at large banks (since hedge funds do not provide this information), about USD 1 trillion of the market value of securities of the hedge fund industry was rehypothecated, as of end-2007.\(^{12}\)
<table>
<thead>
<tr>
<th></th>
<th>Open exchange</th>
<th>OTC, intermediary</th>
<th>OTC bilateral</th>
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<tbody>
<tr>
<td>Size: producers $\mu$</td>
<td>$\mu$</td>
<td>$\mu$</td>
<td>$(1 - \mu)$</td>
</tr>
<tr>
<td>intermediary $\lambda$</td>
<td>$\lambda \mu$</td>
<td>$(1 - \lambda) \mu$</td>
<td>$(1 - \mu)$</td>
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<tr>
<td>trading volume</td>
<td>revealed</td>
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<tr>
<td>Price</td>
<td>$F$</td>
<td>$F$</td>
<td>$F_{B,i}$</td>
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<tr>
<td>traders information set</td>
<td>${F, \lambda \mu X}$</td>
<td>-</td>
<td>$X$</td>
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<tr>
<td>Fee paid by producer</td>
<td>$c_{fee}FX$</td>
<td>$c_{fee}F_{B,i}X$</td>
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<tr>
<td>Fees collected by intermediary</td>
<td>-</td>
<td>$(1 - \lambda) \mu c_{fee}FX$</td>
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Table 1: The Structure of the financial system

Traders in the futures market observe futures price and total hedging demand on the open exchange, $X_{open}$, update their belief regarding spot price. In equilibrium futures price clears the market, expected return equals $r$, and hedging supply equals hedging demand $X_{open}$:

$$E \left[ \frac{P - F}{F} | F, X_{open} \right] = r$$

Traders expect to get $E [P - F | F, X_{open}]$ multiplied by 1 from participating in the future market, but by doing this he would have to lose $Fr$ units of consumption good due to the margin requirements - opportunity cost. His one unit of consumption good is not invested in the capital market, but lies as a collateral on a brokerage account, thus losing $Fr = F\tilde{c}r$, where we assume that $\tilde{c}$ fraction of notional amount $F \cdot 1$ that should consists the collateral, and $\tilde{r}$ - market interest rate. If $\tilde{c} = \tilde{c}$, so that margin requirements are the same for producers and traders, then $r = c_{fee}$. But for now we distinguish the two.

Traders must make inference regarding spot price based on futures price they observe. Spot price will be defined by the total volume of production and therefore total hedging volume. Unfortunately for traders only a fraction of total volumes is traded on the open exchange and, hence, observed. Number of contracts trading on the open exchange is $X_{open}$ and consists of fraction $\lambda \mu$ of total production volume $X$. We will show later that $X_{open}$ and $F$ are informationally equivalent, hence we drop $F$ from information set.

$$X_{open} = \lambda \mu X$$

Putting it back and using $P = \xi X^{-\rho}$

$$F(1 + r) = E[\xi X^{-\rho} | X_{open}]$$

Hence, using $X = \frac{F(1 - c_{fee})}{\gamma}$, $X^{-\rho} = \frac{X_{open}^{-\rho}}{(\lambda \mu \nu)^{-\rho}}$

$$F = \frac{\xi}{1 + r} X_{open}^{-\rho} E[(\lambda \mu \nu)^{\rho} | X_{open}]$$

$$= \frac{\xi}{1 + r} X^{-\rho} E[(\lambda \mu \nu)^{\rho} | X_{open}]$$

$$= \frac{\xi}{1 + r} F^{-\rho} (1 - c_{fee})^{-\rho} E[(\lambda \mu \nu)^{\rho} | X_{open}]$$
Finally, 

\[ F_{1+\rho} = \frac{\tilde{\xi}}{1 + r (1 - c_{fee})^\rho} E[ (\lambda \mu \nu)^\rho | X_{open} ] \]

### 3.3.2 Bilateral exchange.

Now let’s consider bilateral OTC market. We assume that after production is fixed, each producer meets one trader bilaterally. That might be a plant or a utility located in the same neighborhood or similar physical counterparty. Or just a financial deal between two players bilaterally, hence more information is available about conditions of each other. Hence, each match would be defined by a random productivity, or degree of fitness. We assume that \( \beta_i \sim U[0, 1 + \bar{\beta}] \). Some producers represent established firms, has good credit history, or are conveniently located with good access to local pipelines. Hence, any trader can easily find capital to hedge production of such producer and costs are lower than \( r \). Whereas some unknown small companies can be perceived risky and hence credit will be more expensive, it can be the case that \( \beta_i \) is greater than one. Later we will show that \( \bar{\beta} \) defines the profit that the intermediary is able to get in the perfect information case.

Also, because all producers are the same, and in the bilateral market traders observe the volume of production of their clients, bilateral traders are able to extrapolate and hence perfectly assess the total volume of production. Altogether that gives

\[
E \left[ \frac{P - F_{B,i}}{F_{B,i}} | X \right] = \beta_i r \\
F_{B,i} (1 + \beta_i r) = E[P | X]
\]

As before inference must be made about \( P = \xi X^{-\rho} \), but on bilateral market trader observes \( X \), hence

\[
F_{B,i} = \frac{\tilde{\xi}}{1 + \beta_i r} X^{-\rho}
\]

### 3.3.3 Endogenous demand for open exchange contracts

Now for each producer we may compare the open exchange price \( F \) and available bilateral price given realization \( \beta_i \). The ration of two prices equals

\[
\frac{F}{F_{B,i}} = \frac{1 + \beta_i r E[ (\lambda \mu \nu)^\rho | X_{open} ]}{1 + r (\lambda \mu \nu)^\rho}
\]

Let’s denote \( I(\lambda) \) information mistake done by the market in state in which \( \lambda \) is chosen from \( \lambda^* = \{ \lambda_H, \lambda_L \} \) - optimal schedule of the intermediary, and it depends on the ability of the market to make inference observing \( X_{open} \)

\[
I(\lambda) = \frac{E[ (\lambda \mu \nu)^\rho | X_{open} ]}{(\lambda \mu \nu)^\rho}
\]

and using that

\[
\frac{F}{F_{B,i}} = \frac{1 + \beta_i r I(\lambda)}{1 + r I(\lambda)}
\]

Hence, futures price \( F \) is higher than \( F_{B,i} \) if the market overestimates the spot price based on imperfect information so that information mistake \( I(\lambda) \) is high, or when the productivity of the match is bad - \( \beta_i \) is high. Formally, open exchange is preferred when \( F > F_{B,i} \) or when
\[
\frac{1 + \beta_i r}{1 + r} > \frac{1}{I(\lambda)}
\]

\[
\beta_i > \frac{1 + r}{rI(\lambda)} - \frac{1}{r}
\]

Using \(\beta_i \sim U[0, 1 + \bar\beta]\) we can find \(\mu\)-fraction of producers that gets bad matching shock and prefers to go to open exchange

\[
\mu(\lambda) = \frac{1 + \beta - \frac{1 + r}{rI(\lambda)} + \frac{1}{r}}{1 + \beta} = \frac{1 + \frac{1}{r}}{1 + \beta} \left( \frac{\bar\beta}{1 + \frac{1}{r}} + 1 - \frac{1}{I(\lambda)} \right)
\]

\[
\mu(\lambda) = D \left( \bar\mu + 1 - \frac{1}{I(\lambda)} \right)
\]

where \(D = \frac{1 + \frac{1}{r}}{1 + \bar\beta} \bar\mu = \frac{\bar\beta}{1 + \frac{1}{r}}\), hence, \(\bar\beta\) defines the fraction of producer that choose open exchange even in the absence of information revelation problems.

### 3.4 Intermediary

As was mentioned before, the intermediary offers to a \((1 - \lambda)\) fraction of \(\mu\) producers who picked open exchange a new intermediated contract and keeps the collateral return, \(c_{fee} FX\). Hence, expected total profit of the intermediary in consumption units

\[
U_{intermediary} = \pi \mu_H(1 - \lambda_H)c_{fee}\nu_H F_H X_H + (1 - \pi)\mu_L(1 - \lambda_L)c_{fee}\nu_L F_L X_L
\]

where we assumed linear utility.

Intermediary behaves strategically, choosing schedule \((\lambda_H, \lambda_L)\), facing market expectations \(E[(\lambda\mu\nu)^\rho|X_{open}]\) and knowing the structure of the economy. We are focusing on PBE, so given fixed beliefs of the market, the intermediary’s equilibrium choice should be optimal in each state separately.

### 4 First characterization of equilibrium: production and prices

\[
F^{1+\rho} = \frac{\bar\xi}{1 + r(1 - c_{fee})^\rho} \frac{\gamma^\rho}{E[(\lambda\mu\nu)^\rho|X_{open}]} (\lambda\mu\nu)^\rho
\]

\[
X = \frac{F(1 - c_{fee})}{\gamma}
\]

We are comparing pooling equilibrium in which futures price is non revealing with perfect information case.
4.1 Perfect information equilibrium

Market observes both realization of $\gamma$ and actions of the intermediary. Then $I(\lambda) = E[(\lambda\mu\nu)^\rho|X_{\text{open}}] = 1$ in each state. Hence, futures prices equal

$$F_{H}^{1+\rho} = \frac{\bar{\xi} \gamma_H}{1+r(1-c_{fee})^\rho}; \quad F_{L}^{1+\rho} = \frac{\bar{\xi} \gamma_L}{1+r(1-c_{fee})^\rho}$$

And production in each state

$$X_{H}^{1+\rho} = \frac{\bar{\xi} (1-c_{fee})}{\gamma_H}; \quad X_{L}^{1+\rho} = \frac{\bar{\xi} (1-c_{fee})}{\gamma_L}$$

Hedging intensity:

$$1 - \nu_H = \max\left\{\frac{\bar{\xi}X_H^\rho - F_H(1-c_{fee})}{\psi \sigma^2 \xi^2 X_H^{-2\rho} (1-r)}, 0\right\}; \quad 1 - \nu_L = \max\left\{\frac{\bar{\xi}X_L^\rho - F_L(1-c_{fee})}{\psi \sigma^2 \xi^2 X_L^{-2\rho}}, 0\right\}$$

Finally,

$$\mu_H = \mu_L = D\bar{\mu}$$

Spot price unconditional volatility

$$\text{Var}P = \pi \sigma^2 \xi X_H^{-2\rho} + (1-\pi) \sigma^2 \xi X_L^{-2\rho} = \sigma^2 \xi \left(\frac{\bar{\xi}}{1+r(1-c_{fee})}\right)^{\frac{2\rho}{1-\rho}} \left(\pi \gamma_H \xi^2 + (1-\pi) \gamma_L \xi^2\right)$$

**Corollary 1:** In a perfect info equilibrium a) production is larger in high productivity state: $X_H < X_L$; b) hedging is more intensive in a high productivity state and speculation restriction is not binding: $\nu_H < \nu_L < 1$. a) Use $\gamma_H > \gamma_L$.

b) $\frac{\bar{\xi}X^\rho - F(1-c_{fee})}{X^{1-2\rho}} = \gamma \frac{\pi}{\rho} \xi \frac{2\rho}{1-\rho} \left(1 - \frac{1}{1+r}\right)^{\frac{2\rho}{1-\rho}} (1-c_{fee})^{\frac{2\rho}{1-\rho}} - 1 (r + c_{fee}) \equiv g(\gamma) > 0$

Because $\frac{1}{1-\rho} > 0$, $g(\gamma)$ is an increasing function of $\gamma$ and is always positive. Therefore, $1 - \nu_H > 1 - \nu_L$ and equivalently hedging is more intensive in a high productivity state (low cost state), $\nu_H < \nu_L < 1$. Notice that in a perfect info equilibrium, hedging is never full.

4.2 Pooling equilibrium

In a pooling equilibrium futures price must be non-revealing. In that case traders would have to use prior probabilities to price futures. Non-revelation requires that futures prices are the same in both states, therefore using expression for futures prices we get the following necessary condition:

$$\frac{\gamma_H^\rho}{(\lambda_H\mu_H\nu_H)^\rho} = \frac{\gamma_L^\rho}{(\lambda_L\mu_L\nu_L)^\rho}$$
Using that we can find the mistake that market makes in each state

\[ I_H = \pi + (1 - \pi)^2 \frac{\gamma_H^p}{\gamma_L^p}; \quad I_L = \pi \frac{\gamma_H^p}{\gamma_L^p} + (1 - \pi) \]

because \( z \equiv \frac{\gamma^p}{(\lambda \mu \nu)^p} \) is sufficient statistics. Therefore, pooling price equals

\[ F^{1+p} = \frac{\xi}{1 + r (1 - c_{fee} \xi^p)} (\pi \gamma_H^p + (1 - \pi) \gamma_L^p) \]

Production in each state

\[ X_H^{1+p} = (1 - c_{fee}) \frac{\xi}{1 + r} \left( \frac{\gamma_H^p}{\gamma_L^p} \right); \quad X_L^{1+p} = (1 - c_{fee}) \frac{\xi}{1 + r} \left( \frac{\gamma_H^p}{\gamma_L^p} \right) \]

Hedging intensity:

\[ 1 - \nu_H = \max \left\{ \frac{\xi X_H^{-\rho} - F_H (1 - c_{fee})}{\psi \sigma^2 \xi X_H^{-2\rho}}, 0 \right\}; \quad 1 - \nu_L = \max \left\{ \frac{\xi X_L^{-\rho} - F_L (1 - c_{fee})}{\psi \sigma^2 \xi X_L^{-2\rho}}, 0 \right\} \]

Finally, fractions of producers choosing open exchange in each state are defined by

\[ \mu_H = D \left( \bar{\mu} - (1 - \pi) \frac{\gamma_H^p - \gamma_L^p}{\pi \gamma_H^p + (1 - \pi) \gamma_L^p} \right); \quad \mu_L = D \left( \bar{\mu} + \pi \frac{\gamma_H^p - \gamma_L^p}{\pi \gamma_H^p + (1 - \pi) \gamma_L^p} \right) \]

Spot price unconditional volatility

\[ \text{Var} P = \pi \sigma^2 \xi X_H^{-2\rho} + (1 - \pi) \sigma^2 \xi X_L^{-2\rho} \]

\[ = \sigma^2 \xi \left( \frac{\xi}{1 + r (1 - c_{fee})} \right)^{\frac{-2\rho}{1 + r}} \left( \pi \frac{\gamma_H^p}{\gamma_L^p} \right)^{\frac{-2\rho}{1 + r}} + (1 - \pi) \left( \frac{\gamma_H^p}{\gamma_L^p} \right)^{\frac{-2\rho}{1 + r}} \]

Notice, that “real” variables, such as production volumes, prices, utilities of producers and consumers, distribution of producers across markets are the same in every pooling equilibrium.

**Corollary 2:** In a pooling equilibrium

a) production is larger in high productivity state: \( X_H < X_L \);

b) hedging is more intensive in a high productivity state: \( \nu_H < \nu_L \). a) Use \( \gamma_H > \gamma_L \).

b) Futures price is fixed, \( F_H = F_L \equiv F \), production increases, \( X_H < X_L \). Assume that \( \xi X_L^{-\rho} - F (1 - c_{fee}) > 0 \), hence for all \( X \in [X_H, X_L], \xi X^{-\rho} - F (1 - c_{fee}) > 0 \) and

\[ \left( \frac{\xi X^{-\rho} - F (1 - c_{fee})}{X^{1-2\rho}} \right)' = -\rho \xi X^{-\rho - 1} X^{1-2\rho} - (1 - 2\rho) X^{-2\rho} (\xi X^{-\rho} - F (1 - c_{fee})) \]

\[ = \frac{-\rho \xi X^{-\rho} (1 - \rho) X^{-\rho} + (1 - 2\rho) F (1 - c_{fee})}{X^{2-4\rho}} \]

\[ = \frac{-\rho \xi X^{-\rho} + (1 - 2\rho) F (1 - c_{fee})}{X^{2-4\rho}} \]

\[ < -\rho \frac{\xi X^{-\rho}}{X^{2-4\rho}} < 0 \]

Therefore, \( 1 - \nu_H > 1 - \nu_L \) or equivalently hedging is more intensive in a low productivity state, \( \nu_H < \nu_L \). However, in a pooling equilibrium speculation restriction might bind so that \( \nu_L = 1 \)
Corollary 3: Futures prices in perfect information equilibrium and in any pooling equilibrium are
such that $F_L < F < F_H$ and $F^{1+\rho} = \pi F_H^{1+\rho} + (1-\pi)F_L^{1+\rho}$; production is such that $X(F_L, \gamma_L) < X(F, \gamma_L)$
and $X(F_H, \gamma_H) > X(F, \gamma_H)$. Trivial, using $F^{1+\rho} = \frac{\bar{\xi}}{1 + \rho(1 - \text{fee})}\pi\gamma_H + (1-\pi)\gamma_L = \pi F_H^{1+\rho} + (1-\pi)F_L^{1+\rho}$.

$$X = \frac{F(1 - \text{fee})}{\gamma}$$

Feedback effect. To see the feedback effect more clearly one can find an expression for futures price itself, not
its function. Remember that

$$F = \frac{\bar{\xi}}{1 + \rho}(X(F, \gamma_H))^{-\rho} E[(\lambda \mu \nu)^\rho | X_{\text{open}}] = \frac{\bar{\xi}}{1 + \rho}(X(F, \gamma_L))^{-\rho} E[(\lambda \mu \nu)^\rho | X_{\text{open}}]$$

To get intuition one can rewrite necessary condition for non revelation as

$$\frac{X(F, \gamma_H)^{-\rho}}{(\lambda H\mu H\nu H)^\rho} = \frac{X(F, \gamma_L)^{-\rho}}{(\lambda L\mu L\nu L)^\rho}$$

Putting it back, $F = \frac{\bar{\xi}}{1 + \rho}(X(F, \gamma_H))^{-\rho} \pi (\lambda H\mu H\nu H)^\rho + (1-\pi)(\lambda L\mu L\nu L)^\rho = \frac{\bar{\xi}}{1 + \rho} \pi X(F, \gamma_H)^{-\rho} + (1-\pi)X(F, \gamma_L)^{-\rho}$. In perfect info case $F_H = \frac{\bar{\xi}}{1 + \rho} (X(F_H, \gamma_H))^{-\rho}$ and $F_L = \frac{\bar{\xi}}{1 + \rho} (X(F_L, \gamma_L))^{-\rho}$, therefore we can replace $\frac{\bar{\xi}}{1 + \rho}$ in the expression above to finally get

$$F = \pi F_H \frac{X(F_H, \gamma_H)^\rho}{X(F, \gamma_H)^\rho} + (1-\pi)F_L \frac{X(F_L, \gamma_L)^\rho}{X(F, \gamma_L)^\rho}$$

Corollary 3 implies that weight are such that $X(F_H, \gamma_H)^\rho > 1$ and $X(F_L, \gamma_L)^\rho < 1$, $F_H > F > F_L$, denoting

$$\tilde{F}_H = F_H \frac{X(F_H, \gamma_H)^\rho}{X(F, \gamma_H)^\rho} > F_H$$

and $\tilde{F}_L < F_L \frac{X(F_L, \gamma_L)^\rho}{X(F, \gamma_L)^\rho}$

$$F = \pi \tilde{F}_H + (1-\pi)\tilde{F}_L$$

Therefore, any pooling equilibrium price represents expectation of perfect info prices, but with different weight
representing endogenous changes in production.

Corollary 4: In any equilibrium there are more producers choosing open exchange in high productivity state than in low productivity state, $\mu_H < \mu_L$ (or $\beta_H < 1$). Moreover, any pooling equilibrium
has positive dispersion in composition of producers: $\mu_H < D\tilde{\mu} < \mu_L$ (or $\beta_H > 1$). In perfect
information case dispersion is zero, $\mu_H = D\tilde{\mu} = \mu_L$ (or $\beta_H = 1$). Trivial: In any pooling equilibrium $\mu_H = D\tilde{\mu} = \mu_L = \beta L$. The dispersion comes from the mistakes that the market makes regarding the fraction of
volume traded on the open exchange.
Corollary 5: Spot price volatility in any pooling equilibrium is higher than in perfect information equilibrium. Define \( \frac{S_H}{S_L} = \frac{\gamma_H}{\gamma_L} = 1 + \epsilon \) and using that let’s rewrite variances of spot price in each equilibrium:

\[
\begin{align*}
\text{Var}P(\text{perfect info}) &= \sigma^2 \left( \frac{\bar{\xi}}{1 + r} (1 - c_{fee}) \right) \frac{-2\rho}{1 + \rho} \left( \pi \gamma_H^{2\rho} + (1 - \pi) \gamma_L^{2\rho} \right) \\
 &= \sigma^2 \left( \frac{\bar{\xi}}{1 + r} (1 - c_{fee}) \right) \frac{-2\rho}{1 + \rho} \gamma_L^{2\rho} \left( \pi (1 + \epsilon) \frac{\gamma_H^{2\rho}}{1 + \rho} + (1 - \pi) \right)
\end{align*}
\]

\[
\begin{align*}
\text{Var}P(\text{pooling}) &= \sigma^2 \left( \frac{\bar{\xi}}{1 + r} (1 - c_{fee}) \right) \frac{-2\rho}{1 + \rho} \left( \pi \gamma_H^{2\rho} + (1 - \pi) \gamma_L^{2\rho} \right) \left( \pi \gamma_H^{\rho} + (1 - \pi) \gamma_L^{\rho} \right) \frac{2\rho}{1 + \rho} \\
 &= \sigma^2 \left( \frac{\bar{\xi}}{1 + r} (1 - c_{fee}) \right) \frac{-2\rho}{1 + \rho} \gamma_L^{2\rho} \left( \pi (1 + \epsilon)^2 + (1 - \pi) \right) \left( 1 + \pi \epsilon \right) \frac{-2\rho}{1 + \rho}
\end{align*}
\]

Therefore, the difference in variances is given by

\[
\text{Var}P(\text{pooling}) - \text{Var}P(\text{perfect info}) = \sigma^2 \left( \frac{\bar{\xi}}{1 + r} (1 - c_{fee}) \right) \frac{-2\rho}{1 + \rho} \gamma_L^{2\rho} \Delta_{\text{volatility}}
\]

where

\[
\Delta_{\text{volatility}} = \left( \pi (1 + \epsilon)^2 + (1 - \pi) \right) \left( 1 + \pi \epsilon \right) \frac{-2\rho}{1 + \rho} - \left( \pi (1 + \epsilon) \frac{\gamma_H^{2\rho}}{1 + \rho} + (1 - \pi) \right)
\]

Now let’s use second order Taylor expansion to find out the sign of \( \Delta_{\text{volatility}} \)

\[
\Delta_{\text{volatility}} = \epsilon^2 \pi (1 - \pi) > 0
\]

Notice, that only terms of second order and larger matter, first order terms are the same. Analysis of second order terms shows that volatility of the spot price is higher in a pooling equilibrium. Finally, what matters is the difference in prices \( \epsilon = \frac{S_H}{S_L} - 1 = \frac{\gamma_H}{\gamma_L} - 1 \) and \( \pi (1 - \pi) \) term. Last term represents the probability of getting extreme realizations and will be discussed later when we compare the welfare of consumers. As for now, higher \( \pi (1 - \pi) \) means higher entropy of the distribution of productivity. Entropy reflects the information needed to describe the realization of the random variable relative to ex ante information - prior probabilities. The highest entropy is achieved at \( \pi = \frac{1}{2} \), so that ex ante both states are equally likely, the minimal when \( \pi = 0 \) or \( \pi = 1 \) so that ex ante is known for sure what state will be realized. In a pooling equilibrium market uses prior probabilities, hence higher entropy means less information that market possess ex ante, hence higher mistake done by the market on average, hence higher volatility of the spot price due to oscillations of production volumes in response to pricing errors through the hedging channel. Because the effect comes only from changes in production, the effect would be of second order.

5 Characterization of possible equilibria: the intermediary

We showed how real variables are defined in all pooling equilibria and in a separating equilibrium. However, we have not described the behavior of the intermediary and distribution of workers across the markets. In order to
characterize the behavior of intermediaries we need to specify market beliefs regarding \((\lambda \mu \nu)^s\), which is a fraction of total volume that is actually observable. The intermediary plays signaling game with the market.

Market only observes \(X_{\text{open}}\) and futures price. Notice that

\[
X_{\text{open}} = \lambda \mu \nu X = \frac{\lambda \mu \nu}{\gamma} F (1 - c_{\text{fee}})
\]

\[
F^{1+\rho} = \frac{\bar{\xi}}{1 + r (1 - c_{\text{fee}})^s} \frac{E[(\lambda \mu \nu)^s | X_{\text{open}}]}{(\lambda \mu \nu)^s}
\]

Therefore, as was mentioned before, observing \(X_{\text{open}}\) or \(F\), or both is informationally equivalent to simply observing \(\frac{\gamma}{\lambda \mu \nu}\) or \(z = \frac{\gamma^s}{(\lambda \mu \nu)^s}\). Therefore, we can define market beliefs as a function of observable \(z\):

\[
\kappa(z) \equiv E[(\lambda \mu \nu)^s | X_{\text{open}}, F] = E[(\lambda \mu \nu)^s | z] = \frac{E[\gamma^s | z]}{z}
\]

We will be looking for a PBE in a signaling game between the monopolistic intermediary and market participants.

5.1 Perfect information

Imagine as before that market observes all the necessary information: realization of \(\gamma\) and actions of the intermediary. Then in each state real variables and distribution of producers among markets will be fixed at \(\mu_H, \mu_L, F_H, X_H, F_L, X_L\) and would not depend on \(\lambda\). Therefore the intermediary would choose \(\lambda_H = \lambda_L = 0\) - collect as much collateral as possible, because there would be no information manipulation issues. So that all of the open market deals would be moved to OTC exchange, and all of the collateral would work efficiently.

5.2 Pooling equilibrium

Let’s start with characterizing the set of pooling equilibria by utility that the intermediary gets, without putting restrictions that PBE require (basically no IC).

5.2.1 Utility set

In any pooling equilibrium the intermediary is restricted in his choice of \(\lambda - s\) by

\[
\frac{\lambda_H \mu_H \nu_H}{\lambda_L \mu_L \nu_L} = \frac{\gamma_H}{\gamma_L}
\]

That restriction comes from the fact that futures prices must be non-revealing. Now because \(\mu_H, \mu_L, \nu_H, \nu_L\) are the same in any pooling equilibrium, the restriction above can be rewritten as

\[
\lambda_H = A \lambda_L
\]

where we denote \(A \equiv \frac{\gamma_H \mu_H \nu_H}{\gamma_L \mu_L \nu_L}\). Using corollary 2 one immediately gets \(A > 1\): \(\gamma_H > \gamma_L\); \(\mu_L > \mu_H\); \(\nu_L > \nu_H\). Therefore, the intermediary’s utility

\[
U_{\text{intermediary}} = \pi \mu_H (1 - A \lambda_L) c_{\text{fee}} \nu_H F_H X_H + (1 - \pi) \mu_L (1 - \lambda_L) c_{\text{fee}} \nu_L F_L X_L
\]

will be higher in pooling equilibrium with smaller \(\lambda_L\), attaining maximum for \(\lambda_L \to 0\).
5.2.2 Market beliefs

Let’s consider the worst possible equilibrium for the intermediary, the one with highest \( \lambda_L \). However, there is a bound on how big \( \lambda_L \) can be, as it is necessarily bounded from above by \( \frac{1}{A} \lambda_H \). Hence, \( \lambda_L \leq \frac{1}{A} \lambda_H \leq \frac{1}{A} \) and in the worst case, \( \lambda_H = 1 \) and \( \lambda_L = \frac{1}{A} \). We need to check that the intermediary would not deviate in each state.

Theorem 1: For existence of worst pooling equilibrium it is sufficient to have
\[
\bar{\mu} - (1-\pi) \frac{\gamma_H^\rho - \gamma_L^\rho}{\pi \gamma_H^\rho + (1-\pi) \gamma_L^\rho} > 0 \quad \text{and} \quad D \left( \bar{\mu} + 1 - \frac{\gamma_H^\rho}{\gamma_L^\rho} \right) \leq 0.
\]
First condition is a necessary condition. We will first prove the necessary condition for existence and then will find conditions for no profitable deviations in each state.

Existence of worst pooling Define \( z^* = \frac{\gamma_H^\rho}{(\lambda_H \mu_H \nu_H)^\rho} = \frac{\gamma_L^\rho}{(\lambda_L \mu_L \nu_L)^\rho} \). If we consider the worst equilibrium, then \( \lambda_H = 1 \); therefore, \( z^* = \frac{\gamma_H^\rho}{\mu_H \nu_H} \) where \( \mu_H \) and \( \nu_H \) were found for any pooling equilibrium above. Because parameters are such that \( \nu_H > 0 \), we do not allow speculation, existence requires
\[
z^* = \frac{\gamma_H^\rho}{D \left( \bar{\mu} - (1-\pi) \frac{\gamma_H^\rho - \gamma_L^\rho}{\pi \gamma_H^\rho + (1-\pi) \gamma_L^\rho} \right)} > 0
\]
or alternatively \( \bar{\mu} - (1-\pi) \frac{\gamma_H^\rho - \gamma_L^\rho}{\pi \gamma_H^\rho + (1-\pi) \gamma_L^\rho} > 0 \).

No deviation in H state Define beliefs as
\[
\kappa_{\text{pooling}}(z) \equiv \begin{cases} 
\frac{\pi \gamma_H^\rho + (1-\pi) \gamma_L^\rho}{\gamma_L^\rho}, & z = z^* \\
\gamma_H^\rho \gamma_L^\rho, & z \neq z^*
\end{cases}
\]

\[
U_{\text{intermediary}}(z^*|\gamma_H, \kappa_{\text{pooling}}(z)) = 0
\]

Imagine that instead of choosing \( z^* \) the intermediary chooses some other \( z \). \( \lambda_H(z) = \frac{\gamma_H^\rho}{\mu_H \nu_H z^\rho} \)

\[
U_{\text{intermediary}}(z|\gamma_H, \kappa_{\text{pooling}}(z)) = \mu_H(z, z^*)(1 - \frac{\gamma_H^\rho}{\mu_H \nu_H z^\rho}) c_{f,e} \nu_H(z, z^*) F_H(z, z^*) X_H(z, z^*)
\]

We need to find \( \mu_H(z, z^*) \) and other variables as function of \( z \), if the intermediary deviates from equilibrium action \( z^* \), \( z = \frac{\gamma_H^\rho}{(\lambda_H(z) \mu_H(z, z^*) \nu_H(z, z^*))^\rho} \)

\[
I_H(z, z^*) = \frac{E[(\lambda \mu \nu)^\rho | z \neq z^*]}{(\lambda_H(z) \mu_H(z, z^*) \nu_H(z, z^*))^\rho} = \frac{\gamma_H^\rho}{z \gamma_H^\rho} = \frac{\gamma_H^\rho}{\gamma_H^\rho}
\]
\[ \mu_H(z, z^*) = D \left( \bar{\mu} + 1 - \frac{\gamma_H}{\gamma_L} \right) \]

\[ F_H(z, z^*)^{1+\rho} = \frac{\bar{\xi} 1 1}{1 + r (1 - c_{fee})^\rho \gamma_H} \]

\[ X_H(z, z^*) = \frac{(1 - c_{fee}) \bar{\xi} 1 1}{\gamma_H} \frac{1}{1 + r (1 - c_{fee})^\rho \gamma_H} \]

\[ 1 - \nu_H (z, z^*) = \max \left\{ \frac{\bar{\xi} X_H^\rho - F_H(1 - c_{fee})}{\psi \sigma^2 X_H^1 - 2^p}, 0 \right\} \]

\[ U_{\text{intermediary}}(z | \gamma_H, \kappa_{\text{pooling}}(z)) = D \left( \bar{\mu} + 1 - \frac{\gamma_L}{\gamma_H} \right) \left( 1 - \frac{\gamma_H}{\mu_H \nu_H z^p} \right) c_{fee} \nu_H(z, z^*) F_H(z, z^*) X_H(z, z^*) \]

Notice that \( \{\mu_H(z, z^*), F_H(z, z^*), X_H(z, z^*), \nu_H(z, z^*)\} \) do not depend on \( z \). Therefore, if \( D \left( \bar{\mu} + 1 - \frac{\gamma_H}{\gamma_L} \right) > 0 \), then highest possible utility will be reached if \( z \to \infty \) (or \( \lambda_H(z) \to 0 \)).

Hence, sufficient condition for not deviation in \( H \) state is \( D \left( \bar{\mu} + 1 - \frac{\gamma_H}{\gamma_L} \right) \leq 0 \) or \( \mu_H(z, \kappa_{\text{pooling}}(z)) \leq 0 \).

No deviation in \( L \) state  Similarly,

\[ U_{\text{intermediary}}(z^* | \gamma_L, \kappa_{\text{pooling}}(z)) = \mu_L (1 - \lambda_L) c_{fee} \nu_L F_L X_L \]

\[ F_L^{1+\rho} = \frac{\bar{\xi} 1}{1 + r (1 - c_{fee})^\rho} \left( \pi \gamma_H + (1 - \pi) \gamma_L \right) \]

\[ X_L^{1+\rho} = (1 - c_{fee}) \frac{\bar{\xi} \pi \gamma_H + (1 - \pi) \gamma_L}{\gamma_L^{1+\rho}} \]

\[ 1 - \nu_L = \max \left\{ \frac{\bar{\xi} X_L^\rho - F_L(1 - c_{fee})}{\psi \sigma^2 X_L^1 - 2^p}, 0 \right\} \]

\[ \mu_L = D \left( \bar{\mu} + \pi \frac{\gamma_H - \gamma_L}{\pi \gamma_H + (1 - \pi) \gamma_L} \right) \]

Imagine that instead of choosing \( z^* \) the intermediary chooses some other \( z \). \( \lambda_L(z) = \frac{\gamma_L}{\mu_L \nu_L z^p} \)
We need to find \( \mu_H(z, z^*) \) and other variables as function of \( z \), if the intermediary deviates from equilibrium action \( z^*, z = \gamma H \rho L(z, z*) \nu_L(z, z*) \rho L(z, z*) \mu_L(z, z*) \). 

\[
U_{\text{intermediary}}(z|\gamma_L, \kappa_{\text{pooling}}(z)) = \mu_L(z, z^*) \left( 1 - \frac{\gamma_L}{\mu_L \nu_L z^*) \right) c_{\text{fee}} \nu_L(z, z^*) F_L(z, z^*) X_L(z, z^*)
\]

We need to find \( \mu_H(z, z^*) \) and other variables as function of \( z \), if the intermediary deviates from equilibrium action \( z^*, z = \gamma H \rho L(z, z*) \nu_L(z, z*) \rho L(z, z*) \mu_L(z, z*) \). 

\[
I_L(z, z^*) = \frac{E[(\lambda \mu \nu)|z \neq z^*]}{(\lambda L(z) \mu_L(z, z^*) \nu_L(z, z*) \rho L(z, z*) \mu_L(z, z*)} = \frac{\gamma_L^\rho}{z \gamma L^\rho} = 1
\]

\[
\mu_L(z, z^*) = D \mu \rho < \mu L
\]

\[
F_L(z, z^*)^{1+\rho} = \frac{\xi}{1 + r (1 - \rho)} < F_L^{1+\rho}
\]

\[
X_L(z, z^*) = \frac{(1 - \rho c_{\text{fee}})}{\gamma_L} \frac{\xi}{1 + r (1 - \rho)} < X_L^{1+\rho}
\]

And

\[
1 - \nu_L(z, z^*) = \max \left\{ \frac{\xi X_L(z, z^*)^{-\rho} - F_L(z, z^*)(1 - c_{\text{fee}})}{\psi \sigma_\xi^2 X_L(z, z^*)^{1-2\rho}}, 0 \right\} > 1 - \nu_L
\]

As before maximum is reached if \( z \to \infty \), because other function do not depend on \( z \) and

\[
U_{\text{intermediary}}(z = \infty|\gamma_L, \kappa_{\text{pooling}}(z)) = D \mu c_{\text{fee}} \nu_L(z, z^*) F_L(z, z^*) X_L(z, z^*)
\]

\[
< \mu_L c_{\text{fee}} \nu_L F_L X_L
\]

\[
= U_{\text{intermediary}}(z^*|\gamma_L, \kappa_{\text{pooling}}(z))
\]

Everything is greater if no deviation is done: fraction of producers that prefer open exchange, futures price, total production and fraction hedged. Hence, \( L \) type for sure does not want to deviate.

Thus we found the necessary condition for worst pooling equilibrium to exist.

6 Welfare

As was shown before endogeneity of hedging does not affect equilibrium allocations and basically does not play any significant role rather than uniformly affecting expected utility of producers. Therefore, in this section we abstract from endogeneity of hedging and assume that producers have to fully hedge their production, \( \nu = 1 \). That allows us to simplify the welfare comparison of different allocations, however full rigorous analysis for general case of endogenous hedging is performed in appendix.

So we compare welfare in different equilibria: i) perfect information equilibrium; ii) “worst” pooling equilibrium. Consider \( \{ X_H, X_L, F_H, F_L, P_H(\xi), P_L(\xi), \mu_H, \mu_L, \lambda_H, \lambda_L \} \) equilibrium. Expected utilities of all four types of agents should be included: producers, including both those that trade on the open exchange and those that trade on bilateral exchange, consumers, all traders from all 3 markets, and monopolistic intermediary:

\[
W = U_{\text{producers}} + U_{\text{consumers}} + U_{\text{traders}} + U_{\text{intermediary}}
\]
6.1 Non-revelation vs perfect information

We prove the following theorem

Theorem 2: a) Traders on the bilateral market always get zero expected utility both individually and all together as a group. The rest get zero utility individually, but all together as a group get negative expected utility of participating in the futures market in any pooling equilibrium, whereas get zero in perfect information equilibrium. b) Consumers always prefer non-revelation or pooling equilibrium. c) If \( \gamma < 1 - \frac{\gamma_H - \gamma_F}{2(1+\rho)} \) then there exists threshold \( \tilde{\beta} \) (might \( \tilde{\beta} = 0 \)), such that all producers with \( \beta < \tilde{\beta} \) prefer perfect info equilibrium and the rest prefer pooling equilibrium. d) The intermediary prefers pooling equilibrium if \( \bar{\mu} \) is not too big \( \bar{\mu} < \frac{\epsilon}{1 + \frac{\gamma_H}{\gamma_L} (1 + \frac{2}{1+\rho} \epsilon)} \), where \( \epsilon = \frac{\gamma_H}{\gamma_L} - 1 \).

a) Traders individually and as a group Each single trader evaluates his utility conditional on being able to trade or conditional on execution, and he invests only one unit of consumption good, hence for him expected utility equals 0. And it really does not matter whether information is revealed in prices or not, and hence prior probabilities are used. Formally futures price in each state is such that \( F(1 + r) = E[P|X] \) and therefore

\[
U_{\text{trader (perfect info)}} = E[\pi(P_H - (1 + r)F_H) + (1 - \pi)(P_L - (1 + r)F_L)] \\
= E[\pi(\xi X_H^r - \check{\xi} X_H^r) + (1 - \pi)(\xi X_L^r - \check{\xi} X_L^r)] \\
= 0
\]

\[
U_{\text{trader (pooling)}} = E[\pi(P_H - (1 + r)F) + (1 - \pi)(P_L - (1 + r)F)] \\
= E[\pi(\xi X_H^r - E[P]) + (1 - \pi)(\xi X_L^r - E[P])] \\
= E[\pi(\xi X_H^r + (1 - \pi)\xi X_L^r - (\pi \check{\xi} X_H^r + (1 - \pi)\check{\xi} X_L^r)] \\
= 0
\]

So given opportunity to trade each individual trader correctly uses information available to him and futures price reflect that. Each trader expects zero return on his investment of one single unit of consumption good. Unfortunately, although each trader only trades one unit of consumption good, traders as a group trade different amounts in different states. Of course in a perfect information equilibrium it plays no role:

\[
U_{\text{grouptraders (perfect info)}} = E[\pi(P_H - (1 + r)F_H)\mu_H X_H + (1 - \pi)(P_L - (1 + r)F_L)\mu_L X_L] \\
= E[\pi(\xi X_H^r - \check{\xi} X_H^r)\mu_H X_H + (1 - \pi)(\xi X_L^r - \check{\xi} X_L^r)\mu_L X_L] \\
= E[\pi(\xi - \check{\xi})\mu_H X_H^1 + (1 - \pi)(\xi - \check{\xi})\mu_L X_L^1] \\
= 0
\]

In \( H \) state \( \mu_H X_H \) contracts are traded on open exchange and on OTC through the intermediary, each contract implies that one unit of commodity will be bought on a price \( F_H \) and sold on \( P_H \), margin requirements imply also opportunity cost of \( rF_H \), similarly for the \( L \) state. In the perfect information equilibrium futures price correctly reflects expectation of spot price, accounting for production volume. Therefore, on average all traders together get zero expected utility. But in the case of any pooling equilibrium traders as a group would not get zero utility any longer, instead their profit will be negative if evaluated ex ante!
We can again rewrite utilities as combinations of \( f \) functions. Let’s compare consumer’s utility in any pooling equilibrium with perfect production volume as well, thus they do not make any pricing mistakes. Similar to perfect information case. That is because by trading with one producer they are able to infer the total realized, because \( X_L < X_H \). Therefore, the probability to find himself among the luckiest traders is much smaller than among losers, thus trader should not join the market at first place.

Finally, it is easy to see on the bilateral market all traders get zero expected utility individually and as a group, similar to perfect information case. That is because by trading with one producer they are able to infer the total production volume as well, thus they do not make any pricing mistakes.

\( b) \) Consumers prefer non-revelation Let’s compare consumer’s utility in any pooling equilibrium with perfect information case. We will show that consumers always prefer pooling equilibrium. Using optimality and market clearing conditions, utilities of consumers in two equilibria may be written as

\[
U_{\text{consumers}}(\text{pooling}) = \xi \left[ (1-c_{\text{fee}})(1-\rho) \right] ^{1-\rho} \left( \frac{F^{1-\rho}_{H}}{\gamma^{1-\rho}_{H}} + \frac{(1-\pi)F^{1-\rho}_{L}}{\gamma^{1-\rho}_{L}} \right) \\
U_{\text{consumers}}(\text{perfect info}) = \xi \left[ (1-c_{\text{fee}})(1-\rho) \right] ^{1-\rho} \left( \frac{F^{1-\rho}_{H}}{\gamma^{1-\rho}_{H}} + \frac{(1-\pi)F^{1-\rho}_{L}}{\gamma^{1-\rho}_{L}} \right)
\]

From Corollary 3 we also know that \( F^{1+\rho} = \pi F^{1+\rho}_{H} + (1-\pi)F^{1+\rho}_{L} \). Dropping the constants and denoting \( S \equiv F^{1+\rho} \) we get the following system

\[
S = \pi S_{H} + (1-\pi)S_{L}
\]

Notice that function \( f(S,g) = \frac{\rho}{1-\rho}gS^{\frac{1-\rho}{\rho}} \) has strictly positive cross derivative \( f_{Sg} = \rho S^{\frac{2-\rho}{\rho}} > 0 \) therefore \( f(S,g) \) is supermodular in \( S \) and \( g \). That implies that for any \( g' > g \) and \( S' > S \), \( f(S',g') + f(S,g) > f(S',g) + f(S,g') \). We can again rewrite utilities as combinations of \( f(S,g) \) functions evaluated at \( g_{H} = \frac{1}{\gamma^{1-\rho}_{H}} < \frac{1}{\gamma^{1-\rho}_{L}} = g_{L} \) and \( S_{L} < S < S_{H} \)
that futures price $S$ can be higher pay off than the one in perfect info case.

Finally, supermodularity implies that the sum of extreme payoffs is always preferred to the sum of mixtures, so that $f(S_L, g_L) + f(S_H, g_H) - f(S_H, g_L) - f(S_L, g_H) > 0$ and therefore, by transitivity, pooling equilibrium gives higher payoff than the one in perfect info case.

Figure 1 illustrates the result. Fat green line shows utility of consumer as a function of $S$ given bad productivity $\gamma_H$, fat blue line- given $\gamma_L$. Vertical thin green and blue lines show perfect information equilibrium futures prices in two states: $S_H$ and $S_L$. Thus point $D$ shows utility that consumer gets in perfect info equilibrium in bad productivity state, point $C$ - in good equilibrium state. Vertical black line pins down pooling equilibrium futures price $S = \pi S_H + (1-\pi)S_L$, points $E$ and $B$ show utility of consumer in a pooling equilibrium in two
states. Probability of $\gamma_H$ equals $\pi = 0.5$. Horizontal black line demonstrates the level of utility in pooling equilibrium which is higher than utility in perfect info equilibrium - red line. So because of concavity consumer prefers

\{ $E$ with probability $\pi$; $B$ with probability $(1-\pi)$ \} to

\{ $D$ with $\pi$; $F$ with $(1-\pi)$ \} with $\pi$; \{ $A$ with $\pi$; $C$ with $(1-\pi)$ \} with $(1-\pi)$

But because of supermodularity sum of extremes $A+F$ that gets $\pi(1-\pi)$ weight in total is larger than sum of mixtures $D+C$. And therefore the latter is preferred to perfect information set $\{ D$ with $\pi$; $A$ with $(1-\pi)$ \}.

So pooling equilibrium gives higher utility due to some weight, $\pi(1-\pi)$, assigned to extreme cases instead of the sum of mixtures. The higher that weight the greater would be the difference between the two equilibria. As was mentioned above higher $\pi(1-\pi)$ implies higher entropy and smaller amount of information that market possess ex ante and in a pooling equilibrium, thus larger mistakes it makes.

Result: consumers always achieve higher utility in a pooling equilibrium.

c) Producers prefer non-revelation Given that composition of producers changes form perfect info equilibrium to any pooling equilibrium, consider three types of producers:

i) $\beta_l < \beta_H$ - bilateral in both equilibria and both states;

ii) $\beta_L < \beta_l < 1$ - bilateral in both states in perfect info; open exchange in $L$ and bilateral in $H$ in pooling

iii) $1 < \beta_l < \beta_H$ - open exchange in both states in perfect info; open exchange in $L$ and bilateral in $H$ in pooling

iv) $\beta_H < \beta_l$ - open exchange in both equilibria and both states.

When calculating total utility of producers we need to account for different composition in different states: in a bad productivity state producers that get high beta shock $\beta_l > \beta_H$ choose open exchange, where $\beta_H$ is endogenously defined, that makes $\mu_H = \frac{1+\beta_H-\beta_l}{1+\beta_l}$ number of producers. The fraction $1 - \mu_H$ stays on the bilateral exchange and enjoys their relatively good fitness of the match - $\beta_l \in [0, \beta_H]$, similarly for the good state.
\[ U_{\text{producers}} = \pi \left( \mu_H U_{\text{producer,open}}(\gamma_H) + \frac{1}{1+\beta} \int_0^{\beta_H} U_{\text{producer,bilateral}}(\beta_i; \gamma_H) d\beta_i \right) \]

\[ + (1-\pi) \left( \mu_L U_{\text{producer,open}}(\gamma_L) + \frac{1}{1+\beta} \int_0^{\beta_L} U_{\text{producer,bilateral}}(\beta_i; \gamma_L) d\beta_i \right) \]

Appendix 5 shows that type iv) producers will always prefer pooling equilibrium to perfect information case if
\[ \pi < 1 - \frac{\gamma_H^{1-p}}{\gamma_H^{1-p} + \gamma_L^{1-p}} \cdot \frac{\gamma_H - \gamma_L}{\gamma_H - \gamma_L}. \]
In general the smaller is \( \beta_i \), the smaller are \( \pi \) and \( \epsilon \) the more likely that producer would prefer perfect information equilibrium to pooling equilibrium.

The most “efficient” matches suffer more from a switch from perfect information equilibrium to pooling equilibrium. In a perfect info case in both states they get higher price than the one that market offers due to their high efficiency of the match. But in the pooling equilibrium, that increase relative to market price is smaller in the good productivity state- market price is already high, but they produce quite a lot in that state. Counterparties on the bilateral or physical market observe real production, hence they correctly price futures. Therefore, producer are not able to benefit from market mistake and high futures price in a good productivity state.

In the bilateral market traders know all the information, therefore they correctly price futures contracts and always get zero expected utility even as a group as well. Which means that producers cannot benefit on their expense. Therefore, the only optimal way for subgroup of bilateral agents - producers+traders- can be planner’s allocation given full information - perfect info case. But again that is true because \( \beta_i \) is so small to always guarantee the producer the good offer on the bilateral exchange.

However, when producer trades on the open exchange market at least in one of the two states in a pooling equilibrium the situation changes. Because now he can benefit at the sake of traders from their inability to infer information and price futures. Hence, the threshold \( \tilde{\beta} \) exists such that all producers with \( \beta_i \leq \tilde{\beta} \) prefer perfect information equilibrium and \( \beta_i > \tilde{\beta} \) prefer pooling equilibrium.

d) Monoplistic intermediary Monoplistic intermediary can increase his profit from two things i) getting more producers; ii) getting larger trading volume from each producer.

\[ U_{\text{intermediary}}(\text{perfect info}) = \pi D \bar{\mu} c_{fee} F_H X_H + (1-\pi) D \bar{\mu} c_{fee} F_L X_L \]

\[ = Dc_{fee} (1-c_{fee}) \bar{\mu} \left[ \frac{F_H^2}{\gamma_H^{1-p}} + (1-\pi) \frac{F_L^2}{\gamma_L^{1-p}} \right] \]

\[ = Dc_{fee} (1-c_{fee}) \bar{\mu} \left[ \frac{1}{\gamma_L^{1-p}} (1-\pi) \left( \frac{\pi S_H^{\beta_{fee}} \gamma_H}{(1-\pi) S_L^{\beta_{fee}} \gamma_L} + 1 \right) \right] \]

Intermediary in the worst equilibrium chooses \( \lambda_H = 1 \) and \( \lambda_L = \frac{1}{\bar{A}} = \frac{\mu_H}{\mu_L} \)

\[ U_{\text{intermediary}}(\text{pooling}) = (1-\pi) \mu_L (1-\frac{\mu_H}{\mu_L}) c_{fee} F_X L \]

\[ = c_{fee} (1-c_{fee}) (\mu_L - \mu_H) (1-\pi) \frac{F_L^2}{\gamma_L} \]

\[ = Dc_{fee} (1-c_{fee}) \bar{\mu} \gamma_H^{1-p} \gamma_L^{1-p} \left( \frac{S_H^{\beta_{fee}} \gamma_H}{\pi \gamma_H^{1-p} + (1-\pi) \gamma_L^{1-p}} (1-\pi) \right) \frac{1}{\gamma_L^{1-p}} \left( \frac{\pi S_H^{\beta_{fee}}}{(1-\pi) S_L^{\beta_{fee}} \gamma_H} + 1 \right)^{2\beta_{fee}} \]
\[
U_{\text{intermediary (pooling)}} - U_{\text{intermediary (perfect info)}} = Dc_{\text{fee}}(1 - c_{\text{fee}})S_L^2 \left(1 - \pi \right) \frac{1}{\gamma_L} \Delta_{\text{intermediary}}
\]

\[
\Delta_{\text{intermediary}} = \frac{\gamma_H - \gamma_L}{\pi \gamma_H + (1 - \pi) \gamma_L} \left( \frac{\pi S_H}{S_L} + (1 - \pi) \right) \frac{\epsilon}{\gamma_H} - \bar{\mu} \left[ \frac{\pi S_H^2 \gamma_L}{(1 - \pi) S_L^{\frac{2}{1 + \rho}} \gamma_H} + 1 \right]
\]

\[
= (S_H/S_L - 1) \left( \frac{\pi S_H}{S_L} + (1 - \pi) \right)^{\frac{\epsilon}{\gamma_H}} - \bar{\mu} \left[ \frac{\pi \gamma_L S_H^{2\rho}}{(1 - \pi) \gamma_H S_L^{\frac{2}{1 + \rho}}} + 1 \right]
\]

\[
> \epsilon - \bar{\mu} \left[ \frac{\pi \gamma_L}{(1 - \pi) \gamma_H} \left( 1 + \frac{2}{1 + \rho} \epsilon \right) + \frac{1 - \rho}{1 + \rho (1 + \rho)} \right] + 1
\]

First order expansion shows that \( \Delta_{\text{intermediary}} > 0 \), pooling is preferred if

\[
\bar{\mu} < \frac{\epsilon}{1 + \frac{\pi \gamma_L}{(1 - \pi) \gamma_H} \left( 1 + \frac{2}{1 + \rho} \epsilon \right)}
\]

High \( \bar{\mu} \) implies that the intermediary gets sufficient number of producers to intermediate in perfect information case, that makes a switch to pooling less attractive, because when \( H \) state is realized the intermediary gets nothing at all, losing \( D \bar{\mu} \). On the other hand, high \( \epsilon \) implies that there is a sufficient difference in between the two states, hence the intermediary can get really high profit in high productivity state by disguising. If \( \epsilon \) is close to zero, there is no way of masking the production volume. Higher \( \rho \) means higher curvature of production volumes and again more opportunities to mask volumes.

**Corollary 6:** Worst pooling equilibrium is more likely to be preferred by monopolistic intermediary i) if spot market is inelastic - \( \rho \) is high; ii) if probability of bad productivity, \( \pi \), is low; iii) if distribution of efficiency of producers on the bilateral market is shrunk close to 0 - \( \beta \) is low. Same comparative statics is true for the total welfare as well. Both producers and the intermediary in a pooling equilibrium lose if \( \beta \) and, hence, \( \bar{\mu} \) increases. Producers become less efficient on the bilateral market, and the intermediary gets smaller utility \( (D = \frac{1 + \frac{1}{\beta}}{1 + \beta} \text{ decreases}) \).

In the perfect info equilibrium higher \( \bar{\beta} \) means that \( \bar{\mu} \) is higher, so that in both states the fraction of producers choosing open exchange and thus being ready to accept intermediation services is the same and increases with \( \bar{\beta} \). Hence, higher \( \bar{\beta} \) implies larger utility of the intermediary in perfect info equilibrium.

In the worst pooling equilibrium the intermediary gets utility only in a good productivity state, \( \gamma_L \). Utility of the intermediary depends on i) number of producers that choose open exchange; ii) fraction of them that get a new offer, intermediated contract - that thing is chosen strategically given market beliefs. As \( \bar{\beta} \) increases \( \lambda_L = \frac{\mu_H}{\bar{\mu}} = \frac{\bar{\mu} - (1 - \pi) J}{\bar{\mu} + \pi J} = 1 + \frac{J}{\bar{\mu} + \pi J} \) decreases, thus letting producer to get fraction of a pie. But the size of the pie itself decreases - \( \frac{d \mu_L}{d \beta} < 0 \); \( \frac{d \mu_L}{d \beta} \propto \frac{(1 + \frac{1}{\beta})^{-1} (1 + \bar{\beta}) - (1 + \frac{1}{\beta})^{-1} \bar{\beta} - \pi J}{(1 + \bar{\beta})^2} = \frac{(1 + \frac{1}{\beta})^{-1} - \pi J}{(1 + \bar{\beta})^2} \); 

\[
\pi \frac{\gamma_H - \gamma_L}{\pi \gamma_H + (1 - \pi) \gamma_L} = \pi \epsilon = \left( 1 + \frac{1}{\pi \epsilon} \right)^{-1}, \text{ and because in general } r < \pi \epsilon, \left( 1 + \frac{1}{\pi \epsilon} \right)^{-1} < \left( 1 + \frac{1}{\pi \epsilon} \right) = \pi J \text{ and}
\]

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Figure 2: As $\bar{\beta}$ increases the intermediary is more likely to prefer the perfect info equilibrium to the worst pooling equilibrium.

$\frac{d\mu_L}{d\beta} < 0$. So an increase in $\bar{\beta}$ in a pooling equilibrium leads to a smaller fraction of producers that choose open exchange, or alternatively the size of the pie decreases. Turns out that overall piece, $\mu_L(1 - \lambda_L)$, decreases and therefore the intermediary gets smaller profit in a pooling equilibrium with higher $\bar{\beta}$.

7 Conclusion

The provision of risk management services to commodity producers seems to be a logical extension of standard lending practices by large financial intermediaries. However, the concentration of information and power in one hands raises concerns on possible conflicts of interests and abuse of power. The paper studies a dual financial market that features coexistence of an open exchange and a bilateral market, and shows that a strategic intermediary may prefer an equilibrium that features: i) non-revelation of realization of aggregate productivity shock; ii) feedback effect from financial sector to real sector; iii) inefficient production decisions; iv) higher systematic volatility. Although for tractability reasons we make a few simplifying assumptions, the results suggest that regulatory intervention can be called for and further research is needed.
Appendix

8.1 A1: Where can \( c_{fee} \) come from?

Assume that producers have to hedge 100% of their production volume. Producers are required to put \( \tilde{c} \) fraction of notional amount of the contract, \( FX \), as collateral. Therefore, given interest rate \( r \) producers lose \( \tilde{c} r FX = c_{fee} FX \) units of consumption good. How is \( c_{fee} \) determined?

When low demand shock is realized, the spot price becomes really high, and producer may experience losses if he delivers the commodity according to his futures contract, because \( F < P \). Therefore, he might be tempted to run away, to default on his obligations. So the amount of collateral can be determined as such that it prevents a run away with a certain probability.

If the producer honors his obligations, he gets

\[
U(\text{no run away}) = F(1 - c_{fee})X - \frac{\gamma X^2}{2}
\]

If instead the producer walks away, he losses the collateral, but enjoys the higher spot price on the market

\[
U(\text{run away}) = P(\xi, x)X - (1 + r)cFX - \frac{\gamma X^2}{2}
\]

\[
= \xi X^{-\rho}X - \frac{(1 + r)}{r} c_{fee} FX - \frac{\gamma X^2}{2}
\]

Hence, no default condition for a particular \( \xi \) is given by

\[
F(1 - c_{fee})X > \xi X^{-\rho}X - \frac{(1 + r)}{r} c_{fee} FX
\]

\[
F(1 + \frac{1}{r} c_{fee}) > \xi X^{-\rho}
\]

\[
c_{fee} \geq \max_{\xi, \gamma} \left\{ \frac{\xi X^{-\rho} - F}{r} \right\}
\]

Therefore, \( c_{fee} \) might be determined based on distribution of \( \xi \) as such that prevents a run away with a certain probability.

8.2 A2: Welfare, endogenous hedging

Reference to Appendix 2, welfare for endogenous hedging

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8.3 A3: Complementarity in production function

Let’s find a condition that would imply complementarity of production function wrt to price and productivity, or supermodularity wrt to \( F \) and \( \frac{1}{\gamma} \). To do that we consider production cost function \( c(\gamma, x) \) and assume that it is differentiable as many times as needed. We are interested in cross derivative \( \frac{d^2 x}{dFd\frac{1}{\gamma}} \). Define profit function as before

\[
U_{\text{producer}} = FX - c(\gamma, x)
\]
First order condition implies \( F - c_x'(\gamma, x) = 0 \). Taking the derivative wrt \( F \), we get \( 1 - c''_{xx}(\gamma, x) \frac{dx}{dF} = 0 \); and then wrt \( \gamma \) to finally get

\[
-c''_{xx\gamma}(\gamma, x) \frac{dx}{dF} - c'''_{xxx}(\gamma, x) \frac{dx}{dF} + c''_{xx}(\gamma, x) \frac{d^2x}{dFd\gamma} = 0
\]

Rearranging gives

\[
\frac{d^2x}{dFd\gamma} = -\frac{c''_{xx\gamma}(\gamma, x) \frac{dx}{dF} + c'''_{xxx}(\gamma, x) \frac{dx}{dF}}{c''_{xx}(\gamma, x) - c''_{xx}(\gamma, x)}
\]

We already have that \( \frac{dx}{dF} = \frac{1}{c''_{xx}(\gamma, x)} \) from differentiating foc wrt \( F \); and differentiation of the foc wrt to \( \gamma \) gives \( \frac{dx}{d\gamma} = -\frac{c''_{xx}(\gamma, x)}{c''_{xx}(\gamma, x)} \) Putting it all together we find that

\[
\frac{d^2x}{dFd\gamma} = -\frac{c''_{xx\gamma}(\gamma, x) - c'''_{xxx}(\gamma, x) c''_{xx}(\gamma, x)}{(c''_{xx}(\gamma, x))^2}
\]

Let’s find the condition that would imply positive cross derivation of production wrt to \( P \) and productivity, \( \frac{1}{\gamma} \). Switching from costs to productivity we get

\[
\frac{d^2x}{dFd\frac{1}{\gamma}} = \frac{c''_{xx\gamma}(\gamma, x) - c'''_{xxx}(\gamma, x) c''_{xx}(\gamma, x)}{(c''_{xx}(\gamma, x))^2}
\]

Therefore, supermodularity requires

\[
c''_{xx\gamma}(\gamma, x) - c'''_{xxx}(\gamma, x) c''_{xx}(\gamma, x) > 0
\]

In our example, \( c(\gamma, x) = \frac{\gamma x^2}{2} \), hence, \( c_x = \gamma x, c_{xx} = \gamma > 0, c_{xxx} = 0, c_{x\gamma} = x > 0, c_{xx\gamma} = 1 > 0 \), therefore the condition is satisfied, \( \frac{d^2x}{dFd\frac{1}{\gamma}} = 1 \). In general if we assume that third derivative is positive, \( c'''_{xxx}(\gamma, x) \geq 0 \), then \( c_{xx\gamma} \) must be sufficiently positive in order to get SM wrt to \( F \) and \( \frac{1}{\gamma} \).

### 8.4 A4: Bilateral producers

Given that composition of producers changes form perfect info equilibrium to any pooling equilibrium, consider three types of producers:

i) \( \beta_L < \beta_i \) - bilateral in both equilibria and both states;

ii) \( \beta_L < \beta_i < 1 \) - bilateral in both states in perfect info; open exchange in \( L \) and bilateral in \( H \) in pooling

iii) \( 1 < \beta_i < \beta_H \) - open exchange in both states in perfect info; open exchange in \( L \) and bilateral in \( H \) in pooling

iv) \( \beta_H < \beta_i \) - open exchange in both equilibria and both states.
First let’s consider the simplest case - iv) producers that trade on the open exchange always. For them similar logic regarding supermodularity could be applied, but Jensen’s inequality cannot, hence it is harder to prove the same result. Instead we use Taylor expansions. Rewritten utilities, dropping the constant \((1-c_{fe})^2\frac{1}{2}i\):

\[
U_{p, open}(pooling) = \left(\frac{\pi}{\gamma_H} + (1 - \pi)\frac{1}{\gamma_L}\right)S^{\frac{2}{1+\rho}}
\]

\[
U_{p, open}(perfect info) = \pi \frac{S^{\frac{2}{1+\rho}}}{\gamma_H} + (1 - \pi)\frac{1}{\gamma_L}S^{\frac{2}{1+\rho}}
\]

Instead of JI we use Taylor expansion around \(S_H = S_L\):

\[
U_{p, open}(pooling) = \left(\frac{\pi}{\gamma_H} + (1 - \pi)\frac{1}{\gamma_L}\right)S^{\frac{2}{1+\rho}}
\]

\[
= \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} + 1\right) \left(\pi S_H + (1 - \pi)S_L\right)^{\frac{2}{1+\rho}}
\]

\[
= \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} + 1\right) \left(\frac{S_H}{S_L} + (1 - \pi)\right)^{\frac{2}{1+\rho}}
\]

\[
= \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} + 1\right) \left(1 + \pi \epsilon \right)^{\frac{2}{1+\rho}}
\]

\[
U_{p, open}(perfect info) = \left(\frac{\pi}{\gamma_H}S_H^{\frac{2}{1+\rho}} + (1 - \pi)\frac{1}{\gamma_L}S_L^{\frac{2}{1+\rho}}\right)
\]

\[
= \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} \left(\frac{S_H}{S_L}\right)^{\frac{2}{1+\rho}} + 1\right)
\]

\[
= \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} \left(1 + \epsilon \right)^{\frac{2}{1+\rho}} + 1\right)
\]

We know that \((1 + \pi \epsilon)^{\frac{2}{1+\rho}} > 1 + \pi \frac{2}{1+\rho} \epsilon\) and \((1 + \epsilon)^{\frac{2}{1+\rho}} < 1 + \frac{2}{1+\rho} \epsilon\) and \(\frac{1 - \rho}{2(1+\rho)^2} \epsilon^2\). Hence, difference in utilities

\[
U_{p, open}(pooling) - U_{p, open}(perfect info) = \frac{1 - \pi}{\gamma_L} \left(\frac{\pi}{1 - \pi \gamma_H} \left(1 + \epsilon \right)^{\frac{2}{1+\rho}} + 1\right)
\]

\[
\Delta > \left(\frac{\pi}{1 - \pi \gamma_H} + 1\right) \left[1 + \frac{2}{1+\rho} \epsilon\right] - \left(\frac{\pi}{1 - \pi \gamma_H} \left[1 + \frac{2}{1+\rho} \epsilon + \frac{1 - \rho}{(1+\rho)^2} \epsilon^2\right] + 1\right)
\]

\[
= \pi \left(1 - \frac{\gamma_L}{\gamma_H} - \frac{1 - \rho}{2(1 + \rho)^2} \epsilon \frac{1}{1 - \pi \gamma_H} \frac{\gamma_L}{\gamma_H} + 1\right)
\]

Pooling equilibrium will be preferred if \(\Delta > 0\), when \(\pi\) is not too large:

\[
\pi < 1 - \frac{\gamma_L^2\epsilon(1 - \rho)}{2(1 + \rho)(\gamma_H - \gamma_L)}
\]
If we consider only first order expansion, then

$$U_p(pooling) - U_p(perfect\ info) = \pi(1 - \pi)S_L^{\frac{2}{1 + \rho}} \frac{1}{\gamma_L} \left(1 - \frac{\gamma_L}{\rho}ight) \frac{2}{1 + \rho} \left(\frac{\gamma_H}{\rho} - 1\right) \left(1 - c_{fee}\right)^2 \frac{1}{2}$$

So again, the difference will be greater if \(\pi(1 - \pi)\) is larger, if extreme allocation are present in the payoff function. Also, the difference increase with larger \(\frac{\gamma_H}{\gamma_L}\).

Now let’s consider the opposite case - i) producers that always trade on the bilateral exchange, \(\beta_i < \beta_L\)  Denote \(A = \frac{1}{1 + \beta_H}\)

$$U_{p,b}(\beta_i, perfect\ info) = \pi \left(F_{H,i}(1 - c_{fee})X_H^* - \frac{\gamma_H X_H^*}{2}\right) + (1 - \pi) \left(F_{L,i}(1 - c_{fee})X_L^* - \frac{\gamma_L X_L^*}{2}\right)
= \frac{(2F_{H,i} - F_H)F_H(1 - c_{fee})^2}{2\gamma_H} + (1 - \pi)\frac{(2F_{L,i} - F_L)F_L(1 - c_{fee})^2}{2\gamma_L}
= [A - 1]U_{p,open}(perfect\ info)$$

$$U_{p,b}(\beta_i, pooling) = \pi \left(F_{H,i}(1 - c_{fee})X_H^* - \frac{\gamma_H X_H^*}{2}\right) + (1 - \pi) \left(F_{L,i}(1 - c_{fee})X_L^* - \frac{\gamma_L X_L^*}{2}\right)
= \frac{\left[A - \frac{1}{I_H - 1}\right] F^2(1 - c_{fee})^2}{2\gamma_H} + (1 - \pi) \frac{\left[A - \frac{1}{I_L - 1}\right] F^2(1 - c_{fee})^2}{2\gamma_L}
= [A - 1]U_{p,open}(pooling) + AF^2(1 - c_{fee})^2 \frac{1}{2} \left(\frac{1}{I_H - 1}\right) + (1 - \pi) \frac{1}{I_L - 1}
= [A - 1]U_{p,open}(pooling) + AS_L^{\frac{2}{1 + \rho}} \left(\pi \frac{S_H}{S_L} + (1 - \pi)\right)^{\frac{2}{1 + \rho}} \left(1 - c_{fee}\right)^2 \frac{1}{2} \pi(1 - \pi) \frac{\gamma_H - \gamma_L}{\pi \gamma_H + (1 - \pi) \gamma_L} \left(\frac{1}{\gamma_H - 1}\right)$$

First order expansion dropping the constant \( (1 - c_{fee})^2 \frac{1}{2} S_L^{\frac{2}{1 + \rho}} \frac{1 - \pi}{\gamma_L} \) as before

$$U_{p,b}(\beta_i, pooling) - U_{p,b}(\beta_i, perfect\ info) 
\propto [A - 1]\pi \left(1 - \frac{\gamma_L}{\gamma_H}\right) \frac{2}{1 + \rho} \epsilon + A \left(\pi \frac{S_H}{S_L} + (1 - \pi)\right) \frac{\gamma_H - \gamma_L}{\pi \gamma_H + (1 - \pi) \gamma_L} \left(\frac{1}{\gamma_H - 1}\right)
= \pi \left[(A - 1) \frac{2}{1 + \rho} - A\right] \epsilon \left(1 - \frac{\gamma_L}{\gamma_H}\right)$$

Pooling is preferred if
\[(A - 1) \frac{2}{1 + \rho} - A > 0 \]
\[2A - 2 - A\rho > 0 \]
\[\rho < \frac{A - 2}{A} = \frac{2 \frac{1 + r}{1 + \beta_i r} - 2}{2 \frac{1 + r}{1 + \beta_i r}} = \frac{(1 - \beta_i)r}{1 + r} \]
\[(1 - \beta_i) > \frac{1 + r}{\rho} \frac{1}{r} \]
\[\beta_i < 1 - \rho \frac{1 + r}{r} \]

However, if \(\rho\) is sufficiently big it is likely to be the case that all producers of i) type with \(\beta_i < \beta_L\) would prefer perfect information case.

\[U_{p,b}(\beta_i, pooling) - U_{p,b}(\beta_i, perfect info) = \pi(1 - \pi)S_L^{\frac{2\gamma_L^2}{L}} \left( (A - 1) \frac{2}{1 + \rho} - A \right) \left( \frac{\gamma_H^2}{\gamma_L^2} - 1 \right) \left( 1 - \frac{\gamma_L}{\gamma_H} \right) (1 - c_{fee})^2 \frac{1}{2} \]

Now let’s consider ii) \(\beta_L < \beta_i < 1\) - bilateral in both states in perfect info; open exchange in \(L\) and bilateral in \(H\) in pooling. As before denote \(A = 2 - \frac{1 + r}{1 + \beta_i r} \)

\[U_{p,b}(\beta_i, perfect info) = \pi \left( F_{H,i}(1 - c_{fee})X_H^* - \frac{\gamma_H X_H^*}{2} \right) + (1 - \pi) \left( F_L(1 - c_{fee})X_L^* - \frac{\gamma_L X_L^*}{2} \right) \]
\[= \pi \frac{(2F_{H,i} - F_H)F_H(1 - c_{fee})^2}{2\gamma_H} + (1 - \pi) \frac{(2F_{L,i} - F_L)F_L(1 - c_{fee})^2}{2\gamma_L} \]
\[= [A - 1]U_{p,open}(perfect info) \]
\[I_H < 1 < I_L \]

\[U_{p,b}(\beta_i, pooling) = \pi \left( F_{H,i}(1 - c_{fee})X_H^* - \frac{\gamma_H X_H^*}{2} \right) + (1 - \pi) \left( F(1 - c_{fee})X_L^* - \frac{\gamma_L X_L^*}{2} \right) \]
\[= \pi \frac{A}{I_H - 1} \left( \frac{1 + r}{L} - 1 \right) F^2(1 - c_{fee})^2 \]
\[+ (1 - \pi) \frac{F^2(1 - c_{fee})^2}{2\gamma_L} \]
\[= [A - 1]U_{p,open}(pooling) + \pi \frac{A}{I_H - 1} \left( \frac{1 + r}{L} - 1 \right) F^2(1 - c_{fee})^2 \]
\[+ (1 - \pi) \frac{F^2(1 - c_{fee})^2}{2\gamma_L} \left( 1 - 2 \frac{1 + r}{1 + \beta_i r} \right) \]

Dropping the constant \((1 - c_{fee})^2 \frac{1}{2} S_L^{\frac{2\gamma_L^2}{L}} \frac{1 - \pi}{\gamma_L} \) and keeping only first order terms.
\[ U_{p,b}(\beta_i, \text{pooling}) - U_{p,b}(\beta_i, \text{perfect info}) \propto [A - 1] \pi \left( 1 - \frac{\gamma_L}{\gamma_H} \right) \frac{2}{1 + \rho} \epsilon + \]

\[
\frac{A \gamma_H^p - \gamma_H^L}{\pi \gamma_H^p + (1 - \pi) \gamma_L^p} \left( \frac{S_H}{S_L} + (1 - \pi) \right) \frac{\epsilon^2}{\gamma_H/\gamma_L} + \left( \frac{S_H}{S_L} + (1 - \pi) \right) \frac{\epsilon^2}{(1 - A)}
\]

\[ = (1 - A) + \pi \epsilon \left( (A - 1) \frac{2}{1 + \rho} - A \right) \left( -\frac{\gamma_L}{\gamma_H} \right)
\]

Therefore, zero order term is negative, \( A = 2 \frac{1 + r}{1 + \beta_i r} > 1 \). First order term is positive if \( A < \frac{2}{1 - \rho} \). As \( \beta_i \) increase, \( A \) decreases and hence the difference is more likely to become positive. When \( \pi \) is close to zero, the difference is more likely to be negative and perfect information case would be preferred.

For a given \( \beta_i \) as \( \epsilon \) (or \( \pi \)) increases that condition becomes easier to satisfy.

Now let’s consider iii) \( 1 < \beta_i < \beta_H - \text{open exchange in both states in perfect info; open exchange in } L \text{ and bilateral in } H \text{ in pooling} \)

\[ U_{p,b}(\beta_i, \text{perfect info}) = U_{p,\text{open}}(\text{perfect info}) \]

\[ I_H < 1 < I_L \]

\[ U_{p,b}(\beta_i, \text{pooling}) = \pi \left( F_{H,i}(1 - c_{fee})X_H^* - \frac{\gamma_H X_H^2}{2} \right) + (1 - \pi) \left( F(1 - c_{fee})X_L^* - \frac{\gamma_L X_L^2}{2} \right) \]

\[ = \pi \left[ \frac{2}{\gamma_H} \left( 1 + r \frac{1}{1 + \beta_i r} I_H - 1 \right) F^2(1 - c_{fee})^2 \right] + (1 - \pi) \frac{F^2(1 - c_{fee})^2}{2\gamma_L} \]

\[ > \pi \frac{F^2(1 - c_{fee})^2}{2\gamma_H} + (1 - \pi) \frac{F^2(1 - c_{fee})^2}{2\gamma_L} \]

\[ = U_{p,\text{open}}(\text{pooling}) \]

Therefore if type iv) prefers pooling, then type ii) prefers it as well.

Results. Using \( U_{p,b}(\beta_i, \text{pooling}) - U_{p,b}(\beta_i, \text{perfect info}) = C \Delta(\beta_i) \), where \( C = (1 - c_{fee})^2 \frac{1}{2} S_H \pi^{\frac{\epsilon^2}{\gamma_L}} \frac{1}{\gamma_L} A = 2 \frac{1 + r}{1 + \beta_i r} \).

\[ \epsilon = \left( \frac{\gamma_H}{\gamma_L} - 1 \right) \]

i)

\[ \Delta(\beta_i) = \pi (1 - \pi) \left( (A - 1) \frac{2}{1 + \rho} - A \right) \left( 1 - \frac{\gamma_L}{\gamma_H} \right) \epsilon \]

ii)

\[ \Delta(\beta_i) = (1 - \pi)(1 - A) + \pi (1 - \pi) \left( (A - 1) \frac{2}{1 + \rho} - A \right) \left( -\frac{\gamma_L}{\gamma_H} \right) \epsilon \]
iv) 

\[ \Delta(\beta_i) = \pi(1 - \pi) \left( 1 - \frac{\gamma_L}{\gamma_H} \right) \frac{2}{1 + \rho} \epsilon \]

Notice that every in each case delta is proportional to \( \pi(1 - \pi) \). Only case \( ii \) has also zero order term.