Dynamic Disclosure with Uncertain Disclosure Costs*

Daniel Bird

Abstract

This paper extends the canonical single-period model of voluntary costly disclosure to a dynamic setting with asymmetric information over disclosure costs. I show that dynamic incentives have an ambiguous effect on firms’ disclosure decision and that the cost of disclosure no longer prevents full disclosure by all firms. My main results show that (1) there is a positive inter-temporal connection between disclosure at different periods, (2) dynamic incentives increase the firms’ level of disclosure if and only if the expected cost of disclosure is below a critical threshold and (3) when the expected cost of disclosure is low enough, dynamic incentives lead firms to use a strategy of full disclosure.

1 Introduction

This paper offers a new model to explore the incentives guiding voluntary disclosure decisions of firms functioning in a complex economic environment, with multiple periods and uncertain disclosure costs, departing from the stylized assumptions of single period and common knowledge of disclosure costs. These stylized assumptions date back to Verrecchia’s (1983) seminal paper, which, in turn, redressed the unrealistic unraveling prediction of Grossman and Hart (1980), Grossman (1981) and Milgrom (1981). Grossman, Hart and Milgrom showed that when firms are known to have verifiable information that they can disclose at no cost, adverse selection leads to full disclosure of the information in

*I am grateful to Eddie Dekel, Ronald Dye, Eti Einhorn and Asher Wolinsky for valuable suggestions and comments. I also appreciate the helpful comments of seminar participants at Northwestern University and Tel-Aviv University.
equilibrium, a prediction contrary to real world experience. Verrecchia added a
known disclosure cost into this setting; his assumption resolved this conundrum
and has since has been a staple assumption in the study of voluntary disclo-
sure,¹ but it concealed other economic forces relevant to understanding firms’
disclosure decisions. In this paper, I show that when a dynamic environment
(including market uncertainty about disclosure costs) is incorporated into the
model, firms use disclosure to signal their disclosure cost in addition to reveal-
ing their current profits, and equilibria with different types of disclosure policies
exist.

In a dynamic environment, when the expected cost of disclosure is low, the
attempts of firms to manipulate market beliefs can lead to an unraveling re-
sult similar to that of Milgrom (1981), albeit for different reasons. Whereas in
Milgrom’s work the unraveling sequence unfolds on the axis of current profit,
in my model it does so on the axis of disclosure costs. Even when unravelling
does not occur, I show that when expected disclosure costs are low, dynamic
incentives increase the probability of disclosure due to the firm’s attempts to
effect market beliefs. Conversely, when expected costs are high, high-cost (and
consequently low-cost) firms have a dynamic incentive to avoid disclosure since
doing so reduces both their current price and their actual liquidation value, in
contrast with the static model where only the latter applies.

The incentives in a dynamic environment largely originate from the implicit
commitment created by current disclosure to disclose in the future, a com-
mitment driven by two separate factors. Firstly, an increase in the market’s
belief that the firm has low disclosure costs leads to harsher judgment of future
non-disclosure which, in turn, stimulates more disclosure. Secondly, the firm’s
liquidation value is reduced by the unknown real cost of past disclosure and,
therefore, the firm has an incentive to prevent further reduction in the firm’s
expected liquidation value by continuing to signal a low cost through further
disclosure. I show that this effect can become the dominant consideration in
later periods driving low-cost firms to use a strategy of full disclosure (if they
disclosed in the first period), a behavior contingent on the inter-temporal dy-

¹For a comprehensive review of this literature see Dye (2001), Healy and Palepu (2001)
and Verrecchia (2001).
The dynamics of disclosure decision. This mechanism highlights a novel link connecting multiple disclosure decisions over time.

A firm may be willing to disclose low profits, in a world with uncertainty on two dimensions (current profit and disclosure costs), if this reduces the market’s beliefs about its disclosure costs and increases its continuation value. In this case, an unravelling process different than described by Milgrom occurs. The motive for full disclosure is signaling high-value to investors, rather than avoiding unfavorable interpretation of non-disclosure. For equilibrium of this type to exist the value of signaling low disclosure costs must outweigh the cost of disclosure (if the firm has any) and of the cost of the implicit commitment to continue disclosing. Since the firm’s actual cost of disclosure affects this trade-off, there are equilibria in which both low-cost and high-cost firms use full disclosure strategies and other equilibria in which only the low-cost firms do so.

The resolution of the trade-off described above determines not only if firms use a strategy of full disclosure or not, but also if dynamic incentives increase disclosure or not. Even when unravelling does not occur, I show that when expected disclosure costs are low the firms’ dynamic incentive to indicate they have low disclosure costs increases the probability of disclosure in both periods, and vice versa. Formally, I identify conditions under which unraveling occurs for one (or all) firms in the first period, and show that dynamic incentives increase disclosure if and only if the proportion of low-cost firms is above a critical threshold.

The model I construct in this paper also shows indirect effects of uncertain disclosure costs in a single period model that have not been previously considered. When the market does not know the firm’s cost of disclosure, it does not know if lack of disclosure is the result of high disclosure costs or low profits. This increases the ability of low-cost firms to conceal low profits and increases their value at the expense of high-cost firms. In addition to transferring value between firms, uncertainty over disclosure costs increases the aggregate cost of disclosure and reduces the aggregate value of firms due to the increase in disclosure by high-cost firms.
1.1 Related Literature

The literature on disclosure of verifiable information originated in reaction to Grossman, Hart and Milgrom’s controversial prediction that adverse selection leads firms to voluntarily disclose all their private information. This unrealistic result hinged on an unraveling argument, namely that among those firms which do not disclose information, the one with the best information has an incentive to separate itself from the others. Subsequently, Verrecchia (1983) precluded full unraveling by assuming a cost associated with disclosure of private information, which implies that once the value of separation from the pool of non-disclosing firms drops below the cost of disclosure the unraveling stops. Dye (1985) further showed that full unraveling can also be prevented by assuming the firm is not always informed about its profit. Given uncertainty regarding the information endowment, the market cannot know whether non-disclosure was a strategic choice and thus evaluates non-disclosure less harshly. Dye (2001) and Verrecchia (2001) comprehensively reviewed the use of these two approaches.

Within this broad body of literature, two papers (Beyer and Dye (2012) and Einhorn and Ziv (2008)) previously dealt with disclosure in a multi-period setting, examining the incentives created by asymmetric information regarding the firm’s type. These two papers assume that firms differ, respectively, in their objectives and information endowments while I assume the difference is in their cost of disclosure. Beyer and Dye (2012) combine the ideas of Dye (1985) with a reputation based model a-la Kreps and Wilson (1982) and analyze a setting in which some managers are uninformed while others are nonstrategic (honest) and disclose their information regardless of its content. Beyer and Dye show that a reputation for being honest is valuable, and that a strategic manager tries to manipulate her reputation by (partially) mimicking an honest manager’s behavior, that is, by disclosing unfavorable information. My paper develops an alternative model with uncertainty over disclosure costs, showing that dynamic incentives have an ambiguous effect on the level of disclosure when firms are assumed to be fully rational. I identify the conditions when Beyer and Dye's (2012) predictions of increased disclosure due to reputational concerns remain valid in the fully rational setting.

Einhorn and Ziv (2008) consider a dynamic model in which firms are profit
maximizing but there is asymmetric information regarding the probability the firm is informed. They assume a positive inter-temporal correlation between the probabilities the firm is informed in various periods, and thus disclosure creates an implicit commitment to continue disclosing in the future. In their model the cost of disclosure is common knowledge, thus commitment reduces the firm’s future profits without providing any benefit, and results in reduced disclosure. By contrast, in my model the cost of disclosure is private information and investors attach a higher value to firms with low disclosure costs. Since the cost of committing to disclose is lower for a firm with low disclosure costs, then if the cost of doing so is not too high, low-cost firms can use this commitment as a means to signal the firm’s cost to investors. Thus, dynamic incentives may increase the probability of disclosure as opposed to the unambiguous prediction of reduced disclosure given by Einhorn and Ziv (2008).

This paper proceeds as follows. Section 2 introduces the model and defines the equilibrium concepts. In section 3, I characterize the equilibrium in a single period model under incomplete information regarding disclosure costs. Section 4 builds on this characterization and analyzes the equilibrium in a two period model, highlighting the effect of dynamic incentives on the probability of disclosure. Section 5 then considers a variant of the model in which the cost of disclosure reduces the utility of the firm’s manager without effecting investors. The final section offers concluding remarks and summarizes the main results. All proofs are relegated to the appendix.

2 The Model

The model I construct extends Verrecchia’s (1983) model of costly disclosure in two directions: it adds asymmetric information regarding the cost of disclosure, and it considers a multi-period environment. In asymmetric information situations, belief over the cost of disclosure determines how the market responds to firms’ non-disclosure. A firm has incentives to influence the market’s beliefs at a given time in order to increase its future profits. Thus, adding asymmetric

---

2The commitment in their model is the result of a mechanism similar to the first of the two mechanisms which generate commitment in my model. The second mechanism does not exist in Einhorn and Ziv’s (2008) setting.
information and a multi-period span to the model can shed light on incentives that have not been studied until now.

Formally, I assume that a firm is active for two periods in each of which it generates a profit of \( q_t \). The profit generated in each period is distributed IID according to a distribution \( G(q) \) with a strictly positive density \( g(q) \) on a closed interval \( [q, \bar{q}] \) for \( 0 \leq q < \bar{q} \leq \infty \). At the beginning of each period, the firm learns its current profit and decides whether to verifiably disclose this information to the market at a cost of \( \tilde{c} \). The cost of disclosure is the firm’s private information, it does not change over time, belongs to the set \( \{0, c\} \), and the prior probability of \( \tilde{c} = 0 \) is given by \( \mu \in (0, 1) \). I further assume that the firm cannot credibly reveal the cost of disclosure, and thus the market ignores any claim the firm makes regarding it.

The literature on voluntary disclosure for the most does not define the exact source of disclosure costs, which is valid in a single period model. In a two-period model with uncertainty over disclosure costs, the cost of disclosure in period 2 may affect the firm’s value in period 1, and thus the source of disclosure costs must be considered. In the main part of this paper, I assume costs borne by the firm, which means that the firm’s expected liquidation value is reduced by expected future disclosure costs: for example, the cost of disclosing proprietary information, and the cost of managerial time that is diverted from profit generating activities to communicating with investors. Notably, these costs affect future (not current) profit. In the final part of this paper I consider the extreme case wherein disclosure costs do not effect the firm’s liquidation value and yet, for one reason or another, they effect the manager’s personal decision to reveal information. I show that this alternative source of disclosure costs has a different quantitative effect on the firm’s disclosure decision.

Regarding timing, I assume the firm distributes its entire profit as dividends at the end of the first period, and is liquidated after the second. This means that disclosure costs in the first period reduce liquidation value and not dividends. The dividends inform the market of the profits in period 1, even if no disclosure was made.\(^3\) However, the incurred disclosure cost is not known

\(^3\)I assume there is no disclosure cost generated by the profits being revealed through the
to the market and must be estimated via inference from equilibrium disclosure strategies. These assumptions are equivalent to assuming that if the firm disclosed information in the first period, the distribution of second period’s profit is shifted down by \( \hat{c} \). In some parts of the paper I use this interpretation to help clarify the narrative.

### 2.1 The Firm’s Disclosure Decision

In each period, the firm has a simple choice, to disclose information – or not. I denote these actions as \( d \) and \( nd \), respectively, and the firm’s chosen action in period \( t \) as \( a_t \). Based on this decision, the market updates its beliefs over the liquidation value of the firm, and prices the firm. The market’s updated belief at time \( t \) over the disclosure costs and profits distribution is denoted by \( \mathcal{I}_t = (\mu, F(q)) \) (in equilibrium the market updates both elements of its belief, based on the firm’s disclosure decision).\(^4\) The firm’s price as function of the market’s beliefs at the first period is denoted by \( p(\mathcal{I}_1) \), and \( L(\mathcal{I}_2) \) denotes the firm’s liquidation value. I assume the firm’s goal is to maximize the sum of its current price and discounted liquidation value. Since the firm does not know its second period profits, and hence its liquidation value, this implies that in the first period the firm maximizes

\[
p(\mathcal{I}_1) + \delta \mathbb{E} L(\mathcal{I}_2)
\]

where \( \delta \in (0,1] \) is the firm’s discount factor.\(^5\) While in the second period it maximizes

\[
L(\mathcal{I}_2)
\]

---

\(^4\)If the firm chooses to disclose information \( F(q) \) is a degenerate distribution centered on the actual profit.

\(^5\)Due to the delayed nature of the disclosure costs I must assume non myopic firms as otherwise firms are not effected by their actual disclosure costs, but only by the market’s belief over these costs.
2.2 The Pricing Function

In order to focus attention on the firm’s disclosure decision I assume a competitive and risk neutral market that does not discount future payments. The price of the firm, then, is the sum of its expected dividends and expected liquidation value minus the expected disclosure costs in the first period if profits were disclosed. The pricing function is

\[ p(I_1) = \begin{cases} 
\mathbb{E}_F(I_1)[q] + \mathbb{E}[(L(I_2)|I_1] & \text{if } a_1 = nd \\
q + \mathbb{E}[(L(I_2)|I_1] - (1 - \mu(I_1))c & \text{if } a_1 = d 
\end{cases} \]

The firm’s liquidation value is the expected value of its second period profits minus the expected cost of disclosure in both period. Therefore the liquidation value is

\[ L(I_2) = \begin{cases} 
q - 2\tilde{c} & \text{if } a_2 = d, a_1 = d \\
q - \tilde{c} & \text{if } a_2 = d, a_1 = nd \\
\mathbb{E}_F(I_2)[q] - c(1 - \mu(I_2)) & \text{if } a_2 = nd, a_1 = d \\
\mathbb{E}_F(I_2)[q] & \text{if } a_2 = nd, a_1 = nd 
\end{cases} \]

The time line of events in each period is

<table>
<thead>
<tr>
<th>Firm learns (q_t)</th>
<th>Market forms (I_t)</th>
<th>Dividends are distributed</th>
<th>Firm chooses (a_t)</th>
<th>Firm is valued</th>
</tr>
</thead>
</table>

2.3 Equilibrium

The solution concept used in this analysis is (weak) Perfect Bayesian Equilibrium. This concept requires that each player chooses an optimal action with regard to their beliefs at all decision nodes, and that beliefs are formed by 6If the market discounts future payments, the delayed disclosure costs obviously incentivize disclosure. Assuming the market does not discount future payments makes it harder to show that dynamic incentives can increase disclosure.
Bayesian updating whenever possible. As the firms are always fully informed, I need only specify the market’s belief at decision nodes that are not along the path of play.

3 Uncertain Disclosure Costs: A Single Period

To understand the effect of asymmetric information over disclosure costs in a dynamic setting it is necessary to first understand the effect of uncertainty in a single period. Conditional on non-disclosure, for any belief held by the market, the firm’s liquidation value is independent of the firm’s type; thus, I can denote this value as $v_{nd}$. As the firm has no incentive to alter the market’s belief about its disclosure costs, it discloses its profits if and only if

$$q - \tilde{c} \geq v_{nd}$$

Consequently, both types of firms use a threshold strategy, and as the firm is indifferent regarding disclosure at the threshold, and non-disclosure provides the same payoff to all firms, the firm’s threshold is linear in its disclosure cost. Finally, the threshold for a firm with no disclosure costs equals $v_{nd}$.

A firm with high disclosure costs, and lowest possible profit, does not disclose its profits and so, conditional on no disclosure, the market’s belief is the result of Bayesian updating

$$v_{nd} = \frac{\mu Pr(nd|\tilde{c} = 0)}{\mu Pr(nd|\tilde{c} = 0) + (1 - \mu) Pr(nd|\tilde{c} = c)} \mathbb{E}(q|nd, \tilde{c} = 0) + \frac{(1 - \mu) Pr(nd|\tilde{c} = c)}{\mu Pr(nd|\tilde{c} = 0) + (1 - \mu) Pr(nd|\tilde{c} = c)} \mathbb{E}(q|nd, \tilde{c} = c)$$

(1)

In equilibrium, a firm discloses information if and only if $q > v_{nd} + \tilde{c}$, thus (1) simplifies to:

$$v_{nd} = \frac{\mu G(v_{nd})}{\mu G(v_{nd}) + (1 - \mu) G(v_{nd} + c)} \mathbb{E}(q|q < v_{nd}) + \frac{(1 - \mu) G(v_{nd} + c)}{\mu G(v_{nd}) + (1 - \mu) G(v_{nd} + c)} \mathbb{E}(q|q < v_{nd} + c)$$

(2)
In addition to providing a simple characterization of equilibrium, equation (2) has multiple economic implications listed in the following proposition

**Proposition 1.**

- **Uncertainty over costs reduces disclosure by firms with low costs and, conversely, increases disclosure by those with high costs.**

- **Uncertainty over costs increases the value of low-cost firms and, conversely, reduces the value of high-cost firms.**

- **Asymmetric information over costs reduces the expected value of firms.**

The intuition behind this proposition is the same idea Dye (1985) used to show that uncertainty regarding information endowment can prevent unraveling. Specifically, a firm with no disclosure costs which would only conceal the lowest profits if its type were known, finds non-disclosure more attractive as it is pooled together with firms with high disclosure costs that do not disclose higher levels of profits. Thus, the expected value of a firm with low disclosure costs is greater under uncertainty, which in turn implies the expected value of a firm with high disclosure costs must decrease due to the market’s rationality. Similarly, high-cost firms disclose more as the pool of non-disclosing firms worsens, which increases the aggregate disclosure costs and reduces aggregate firm value. The aggregate probability of disclosure in the market can increase or decrease depending on the exact form of the distribution function.

### 3.1 The Value Function

For a given second period equilibrium strategy\(^7\) the expected liquidation value for a firm of type \(\hat{c}\) when the market beliefs are \(\mu\) can be represented by a value function

\[
V(\mu, \hat{c}) \equiv G(v_{nd} + \hat{c})v_{nd} + (1 - G(v_{nd} + \hat{c}))(\mathbb{E}(q - \hat{c})|q > v_{nd} + \hat{c}) \tag{3}
\]

The first element of this sum is the value of a firm with undisclosed profits, while the second is the expected value when profits are disclosed. This value

\(^7\)In Appendix B, I provide a sufficient condition for uniqueness of the second period equilibrium.
function is strictly decreasing in $\hat{c}$ as a firm with no disclosure costs can mimic the firm with disclosure costs and optimally discloses profits above $\mathbb{E}[q]$. The value function is also strictly decreasing in $\mu$ as a higher percentage of firms with no disclosure costs reduces the expected value of a firm conditional on no disclosure (which is always the optimal action for a low enough profit).

4 Uncertain Disclosure Costs: Multiple Periods

Armed with the simple representation of the continuation game equation (3) provides, I now start the analysis of the strategic interaction between the firm and the market during the first period. As in most signaling games, different off-path beliefs can be used to sustain multiple equilibria, however, in this paper I focus on equilibria that use threshold strategies. The main reason for doing so is the simplicity of strategies and beliefs required to support threshold equilibria. To support non threshold equilibria the market’s beliefs about disclosure costs must be non-monotonic and discontinuous in the firm’s profit, a feature that does not seem plausible. A more formal justification for focusing on this equilibria is that it is the unique equilibrium that satisfies condition $D.1$ of Cho and Kreps (1987), a selection criteria with a strong intuitive appeal that had been used in similar settings by Einhorn and Ziv (2012). An additional attractive feature of threshold equilibria is that they maximize the disclosure of information. This is the case because non threshold equilibria are supported by harsh off path beliefs after disclosure, beliefs that are ruled out by the $D.1$ condition.\(^8\)

Even though both types of firms use threshold strategies, each firm faces a different tradeoff at its respective threshold. Unsurprisingly, firms with no disclosure costs have a lower disclosure threshold, and by disclosure of these low values, at which high cost firms do not disclose, reveal their type to the market. This revelation has a twofold effect: it reduces the firm’s ability to conceal low profits in the second period, but increases the expected liquidation value by showing that no resources will be diverted to costly disclosure. The

\(^8\)Related papers such as Einhorn and Ziv (2008) and Beyer and Dye (2012) did not require the use of equilibrium refinements to get a unique equilibrium as the existence of uninformed firms that could not disclose any information implied there are no off-path beliefs that could be specified arbitrarily.
gain from uncertainty over disclosure costs is bounded above by the expected disclosure costs incurred by a high-cost firm. And this implies that the gain in expected liquidation value due to revelation is larger than the cost of doing so. Thus by disclosing, a low-cost firm increases its liquidation value but reduces the market’s evaluation of its first period profits.

Firms with high costs, on the other hand, prevent any updating of beliefs by disclosing information, and send an informative, albeit imperfect, signal about their type by no disclosure. Signaling the firm’s high costs also has a twofold effect on liquidation value: it avoids disclosure the market evaluates as potentially costly but increases the market’s belief that the firm has high disclosure costs. The change in the market’s belief also has an unclear effect of the firm’s value, as it reduces the probability of disclosure in the second period and may even result in a reduction in the firm’s expected disclosure costs.

To circumvent this problem and maintain the reasonable assumption that a firm’s value is decreasing in its perceived cost of disclosure, I add assumptions about the distribution of profits and disclosure costs. To achieve this goal and generate a tractable model, I make the strong assumption that the firm’s profits are distributed uniformly, and I normalize the support to \([c, 1 + c]\). Furthermore, I assume \(c < \frac{1}{4}\), so that increasing the cost of disclosure is not valuable for a firm even if it were known to have a high cost of disclosure. Thus, for the rest of this section I assume

\[
G(q) \sim U[c, 1 + c] \\
\]

\(c \in (0, \frac{1}{4})\)

Given this assumption, the value function and disclosure threshold have a simple

---

9The dividends distributed at the end of the first period will reveal the firm’s type and effect is liquidation value. However, when the market prices the firm, it does not know the firm’s type.

10I normalize the lower bound of the support to \(c\) in order to maintain the interpretation that second period profits are positive even when disclosure costs were paid in the first period.

11For a uniform distribution of profits the expected profit of a firm with known disclosure costs of \(c\) is \(E[q] - c(1 - 2c)\).

12Beyer and Dye (2012) also use the assumption that profits are distributed uniformly in order to obtain a tractable model.
closed form solution

\[ v_{nd} = c\sqrt{1 - \mu} + c \]
\[ V(\mu, \tilde{c}) = \frac{1 + 2c}{2} + \frac{(1 - \mu)c^2 + \tilde{c}^2}{2} + \tilde{c}c\sqrt{1 - \mu - \tilde{c}} \]

4.1 The Second Period

In the dynamic setting, the second period’s profit distribution depends on the action taken in the first period. If there was no disclosure, then there are no postponed disclosure costs and both firms draw profits from the same distribution. However, if profits were disclosed in the first period, the market does not know if disclosure costs reduce the distribution of second period profits or not. This implies that by choosing to reveal the profits and cost of disclosure in the second period, a firm with low costs proves to the market that no disclosure costs were paid in the first period. And this, in turn, implies that low-cost firms have an additional incentive to disclose in the second period, an incentive absent in the static model. Revealing the information has the exact opposite effect on firms with high costs. The incentive to avoid disclosure by such firms is further enhanced by the fact that higher probability of disclosure by low-cost firms induces more favorable market’s posterior belief, i.e., a higher probability of high disclosure costs. A-priori, it is unclear how these new contradicting incentives effect the equilibrium following disclosure in the first period. However, the assumption that profits are distributed uniformly implies that in equilibrium low-cost firms would disclose their profits in the second period if they did so in the first. To understand this, recall that a low-cost firm’s disclosure reduces the market’s belief of its incurred cost in the first period by \((1 - \mu)c\). At the same time, in doing so the firm separates itself from the pool of non-disclosing firms, which potentially reduces its value. However, for the most profitable non-disclosing low-cost firm, separating itself from the other firms, creates a loss of less than the difference between disclosure thresholds, \(c\), multiplied by the probability of high disclosure cost \((1 - \mu)\). And this, in turn, implies that for the most profitable low-cost firm separation is valuable. This intuition is formalized by the following lemma

Lemma 1. There is a unique equilibrium following first period disclosure. In this equilibrium each firm acts as if its type were known, i.e. the firm with no
Disclosure costs always discloses, and a firm with high disclosure costs uses a disclosure threshold of $3c$.

Disclosure in the first period implies that in the second period firms act as if their type were known, while after no disclosure there is still uncertainty regarding the actual disclosure costs. Therefore, disclosure in the first period increases aggregate disclosure in the second period if asymmetric information reduces aggregate disclosure. For the uniform distribution asymmetric information over costs decreases the probability of disclosure, thus any incentive increasing disclosure in the first period increases disclosure in the second period as well. This simple observation has an important implication as it states that there is no possible tradeoff between first period and second period disclosure.

**Corollary 1.** The probability of disclosure in the second period is increasing in the probability of first period disclosure.

Lemma 1 also enables us to clarify the expressions for the firm’s price in the first period. After disclosure the expected liquidation value is

$$
\mathbb{E} L_d(\mu) = \mu V(1,0) + (1 - \mu)(V(0,c) - c)
$$

The expected liquidation value after non-disclosure is more complicated, since the market’s beliefs are updated again after the first period dividends are distributed. Denoting the posterior beliefs after observing $q_1$ by $\hat{\mu}$, the liquidation value is

$$
\mathbb{E} L_{nd}(\mu) = \mu \mathbb{E} \hat{\mu}[V(\hat{\mu},0)|\hat{c} = 0] + (1 - \mu) \mathbb{E} \hat{\mu}[V(\hat{\mu},\tilde{c})|\hat{c} = \tilde{c}]
$$

Denoting by $\mu(\cdot)$ the market belief following the firm’s action, the firm’s price after disclosure is

$$
p_d(q) = q + \mathbb{E} L_d(\mu(q))
$$

And, denoting by $q_{nd}$ the expected first period profit conditional on no disclosure, the price is

$$
p_{nd} = q_{nd} + \mathbb{E} L_{nd}(\mu(nd))
$$

**4.2 The First Period**

The firm’s price following disclosure in the first period is not effected by its real disclosure cost, but only by the market’s belief over the cost. However, firms
still have different incentives in the first period, as they maximize the sum of their first period price and the discounted expected liquidation value conditional on their true cost. Since the low-cost firm’s expected liquidation value is greater than the high-cost firm’s expected liquidation value for any market beliefs, and the expected liquidation value is maximized when \( \mu = 0 \), if a high-cost firm discloses the low-cost firm does so as well. This observation is formalized by the following Lemma

**Lemma 2.** The first period disclosure threshold is weakly decreasing in the firm’s cost of disclosure.

### 4.2.1 Full Disclosure by all Firms

In the most extreme case, the incentive to manipulate the market’s belief can become the dominant incentive and lead to an equilibrium with full revelation by all firms. Disclosure of profits that should only be disclosed by a low-cost firm, commits the firm to disclose its profit in the second period (or be evaluated as a firm with the lowest possible profit). This commitment does not reduce the firm’s first period price but only its discounted continuation value. So when the discount factor and the cost of this commitment are low, and especially when the perceived cost of disclosure is low, a high-cost firm might mimic a low-cost firm’s disclosure strategy.\(^{13}\) Consequently, the market’s (on-path) beliefs are not affected by the firm’s disclosure decision, and a standard unraveling argument shows that all low-cost firms must disclose. In such equilibrium, the market learns the firm’s type only in the second period and then decreases the liquidation value accordingly.

**Proposition 2.** Full revelation by all firms is the unique first period equilibrium satisfying condition D.1. if and only if \( \delta \leq \delta_1 \equiv 2\mu(1 - c) - 1 \)

Since \( \delta_1 < 1 \), if firms are patient enough, firms with high disclosure costs do not use a strategy of full disclosure.

### 4.2.2 Full Disclosure by Firms with no Costs

While both firms are willing to commit to disclose in the second period when the discount factor is low enough, for higher discount factors the cost of committing

\[^{13}\text{The cost of this commitment is } V(0, c) - V(1, c) = \frac{3}{2}c^2. \text{ When all firms use a strategy of full disclosure the perceived cost of disclosure is } (1 - \mu)c.\]
to disclose in the second period might be sufficient to deter a high-cost firm from using a strategy of full revelation and making this commitment. For full disclosure to still be an optimal strategy for a low-cost firm, the increase in liquidation value generated by signaling that no disclosure costs will be paid in the future, must outweigh both the reduction in the market’s expectation over $q_1$ and the decrease in the actual liquidation value caused by revealing its type.

In such equilibria, high-cost firm’s will use a strategy of partial disclosure.

**Proposition 3.** Full revelation by a firm with no disclosure costs, and partial revelation by a firm with high disclosure costs, is the unique first period equilibrium satisfying condition D.1. if and only if $\delta_1 < \delta \leq \delta_2 \equiv \frac{4(\mu(1-c)+c)}{2+c}$. Furthermore, if $\delta \in (\delta_1, \delta_2]$ the expected level of disclosure in the first period is greater than the expected level of disclosure in a static model.

This result is stronger than the previous one since it shows that even without discounting the future a firm with no disclosure costs may use a strategy of full disclosure to signal its type. This occurs when the cost of disclosure is small and the prior probability of the firm having no disclosure costs is high. Namely, when uncertainty over disclosure costs does not substantially increase a low-cost firm’s liquidation value and the market’s expectation of first period profits after no disclosure is low.

**Corollary 2.** For any discount factor, a firm with no disclosure costs uses a strategy of full disclosure in the first period profit if

$$\frac{1}{2} < \mu < 1 \text{ and } 0 < c \leq \frac{4\mu - 2}{5 + 4\mu}$$

### 4.2.3 Partial Disclosure by both Firms

When the discount factor is high enough (relative to $\delta_2$) to prevent full disclosure by both firms, firms use a partial disclosure strategy and have non-trivial disclosure thresholds. The equilibrium outcome of the game is then characterized by two indifference equations. The indifference condition for the low-cost firm balances the gain in expected liquidation value from disclosure with the loss in value from revealing first period profits below $q_{nd}$ and the decrease in actual liquidation value. The disclosure threshold for the low-cost firm is the $q_0$ that solves $^{14}$

---

$^{14}$As the market observes $q_1$ before the second period, non-disclosure at the threshold leads to a reversion of the posterior from $\mu_{nd}$ to $\mu$ at the start of the second period.
\[ q_0 + E L_d(1) + \delta V(1, 0) = q_{nd} + E L_{nd}(\mu_{nd}) + \delta V(\mu, 0) \quad (ID_0) \]

The indifference condition for a high-cost firm has the same factors as above with the additional negative effect of the reduction in actual liquidation value following disclosure in the first period. The threshold \( q_c \) solves

\[ q_c + E L_d(\mu) + \delta (V(0, c) - c) = q_{nd} + E L_{nd}(\mu_{nd}) + \delta V(0, c) \quad (ID_c) \]

From these equations it is clear that the level of disclosure is decreasing in the discount factor. The actual liquidation value is higher after non-disclosure, and a higher discount factor increases the weight a firm puts on the actual liquidation value relative to its first period price.\(^{15}\) Therefore, an increase in the discount factor causes firms to raise their disclosure threshold. This increase is augmented by the raise in \( q_{nd} \) resulting from the change in the disclosure strategies of other firms.

**Corollary 3.** The disclosure threshold for both firms is increasing in the discount factor.

As discussed above, dynamic considerations incentivize firms to disclose more information when the perceived cost of disclosure is low. Conversely, when the perceived cost is high, firms with high disclosure costs avoid disclosure since doing so reduces both their current price and their actual liquidation value, in contrast with the static model where only the latter applies. Less disclosure by high-cost firms, in turn, discourages disclosure by low-cost firms and leads to less disclosure by all firms when the perceived cost of doing so is high.

The percentage of low-cost firms (which determines the perceived cost of disclosure) turns out to be the unique factor that determines whether dynamic consideration increase disclosure or not as the following proposition formulates.

**Proposition 4.** For any \( \delta > \delta_2 \) there exists \( \bar{\mu}_\delta(\delta) \in (0, 1) \) such that a firm of type \( \hat{c} \) discloses more in the first period of a dynamic environment than in the static environment if and only if \( \mu > \bar{\mu}_\delta(\delta) \).

\(^{15}\)To see this, note that for a firm with no disclose costs the actual liquidation value after disclosure is \( V(1, 0) \) while after no disclosure it is \( V(\mu, 0) \). While for a firm with high disclosure costs the liquidation value after disclosure is \( V(0, c) - c \) versus \( V(0, c) \) after no disclose.
Combining the previous propositions constitutes the main insight of this paper. In environments with low expected disclosure costs, firms disclose more information in a dynamic setting than in a static one, contrary to the previous results of Einhorn and Ziv (2008). The conflict between these results is due to the difference in the firm’s type space between the two models. In Einhorn and Ziv (2008) the firm’s type is associated with its information endowment, and thus dynamic considerations encourage firms to acquire a reputation for being uninformed that allows them to conceal unfavorable information. Whereas in my model, the firm’s type is associated with the cost of disclosure, and thus dynamic considerations can encourage firms to acquire a reputation for having low disclosure costs in order to increase the expected dividend stream they are likely to generate for investors.

5 Disclosure Costs Borne by the Manager

So far I explored the effect of uncertainty over disclosure costs, based on the plausible assumption that when costs are significant enough to effect the decisions about disclosure they also effect the firm’s value. Bamber, Jiang, and Wang (2010) have empirically shown that a manager’s personal history is relevant in predicting the firm’s disclosure decisions, and thus I consider alternative assumptions as well. In this section, I explore the alternative assumption that disclosure costs are borne by the manager, who makes the disclosure decision, but do not effect the firm’s value. In addition, for ease of exposition I modify the assumption regarding the timing of disclosure costs. I assume that the manager’s disutility from disclosure is incurred not in the following period but immediately.\footnote{This modification is without loss of generality as any future cost, c, which is the certain result of current disclosure, is equivalent to an immediate cost of $\delta c$.}

The source of disclosure costs is not important in a static setting, so the analysis of the second period in section 3 remains valid. Moreover, as the cost of disclosure does not effect investors, the firm’s expected liquidation value is independent of previous disclosure decisions and the market’s belief over the manager’s type (disclosure cost). The firm’s price in the first period, in this
The manager’s utility from disclosure is
\[ q - \bar{c} + \mathbb{E}[q] + \delta V(\mu_d, \bar{c}) \]
and from non-disclosure
\[ q_{nd} + \mathbb{E}[q] + \delta V(\mu_{nd}, \bar{c}) \]
In this alternative, it is impossible to get equilibrium with full disclosure as a manager’s continuation value is decreasing in \( \mu \), and unexpected non-disclosure decreases \( \mu \) due to condition D.1. Thus, equilibrium is characterized by two indifference conditions. For a manager with no disclosure costs, the indifference condition equates the first period gain from disclosing profits above \( q_{nd} \) with the cost of revealing her type and committing to full disclosure in the future
\[ q_0 + \delta V(1, 0) = q_{nd} + \delta V(\mu, 0) \]
For a manager with high disclosure costs, the indifference condition balances the first period gain from disclosing profits above \( q_{nd} \) with the gain from signaling her type through non-disclosure and, thus, reducing her second period expected disclosure costs
\[ q_c - c + \delta V(\mu, c) = q_{nd} + \delta V(0, c) \]
Under this alternative assumption, it is clear that dynamic incentives decrease disclosure as both types prefer to be perceived as managers with high disclosure costs. Managers with actual high disclosure costs are less likely to disclose, and so all managers would imitate their behavior and disclose less information.
Formally

**Proposition 5.** When the cost of disclosure does not effect investors, dynamic incentives reduce disclosure.

### 6 Conclusion and Discussion

Asymmetric information and inter-temporal considerations are commonplace in many settings, yet, to the best of my knowledge, prior to this paper little has been known about how these features effect disclosure decisions. This oversight
is even more surprising due to the fundamental importance of disclosure costs in explaining why firms refrain from disclosure. In this paper, I have tried to rectify this oversight and have offered a model that demonstrates have demonstrated novel implications of asymmetric information over disclosure costs. I have shown that the combination of cost uncertainty and inter-temporal considerations generates incentives effecting the firm’s disclosure decisions.

Firstly, I have shown that in addition to effecting the disclosure decision of a single firm, uncertainty over disclosure costs has general equilibrium implications. Namely, it leads to a transfer of value between firms and increases the value of low-cost firms at the expense of high-cost ones. Furthermore, this transfer increases the deadweight loss created by costly disclosure and reduces the aggregate value of firms. Moreover, as asymmetric information may decrease aggregate disclosure, my work suggests that low disclosure costs may create a larger decreases in aggregate disclosure than was previously thought possible.

Secondly, I have shown that in a two period setting, disclosure costs may fail to prevent full disclosure since lack of disclosure acts as a signal for a low liquidation value. This insight relies on the existence of a known final period which prevents a firm with high disclosure costs from credibly committing to not disclose in the future. However, even without a terminal date, the mechanism studied in this paper may lead to full disclosure by some firms. Moreover, it raises an important question regarding the conditions under which a firm could credibly commit to avoid disclosure in the future, a commitment which when credible, could massively enhance the effect of disclosure costs on disclosure decisions and support equilibria in which different firms use very different disclosure policies.

Thirdly, I have shown that the effect of disclosure costs is enhanced (or weakened) due to its interaction with other mechanisms that effect the firm’s value. Specifically, when disclosure doubles as signaling device for the firm’s type, low cost of signaling reduces the effect disclosure costs have on the disclosure probability and vice-versa. Moreover, I have demonstrated that different assumptions regarding the underlying source of disclosure costs can alter the inter-temporal incentives generated by disclosure costs.
Clearly, this paper merely skims the surface in understanding the mechanisms by which asymmetric information over disclosure costs effects disclosure decisions. However, it shows the importance of doing so, and indicates subsequent questions we need to pursue. I believe that models with an infinite horizon in which the firm’s type may change over time may be especially important to uncovering the full effect of asymmetric information, but I leave pursuing such questions to future work.

Appendix - Proofs

Proof of Proposition 1

As
\[
\frac{(1-\mu)G(v_{nd}+c)}{\mu G(v_{nd}) + (1-\mu)G(v_{nd}+c)} \mathbb{E}(q|q < v_{nd} + c) > q
\]
for any \(v_{nd}\) we must have that \(v_{nd} > q\). Implying the disclosure threshold for a low-cost firm is strictly greater than \(q\), as opposed to the threshold of \(q\) it would have if its type were known. Clearly, this implies uncertainty increases the value of a low-cost firm, as the strategy of full disclosure is available but non optimal.

For a firm with a known disclosure costs of \(c\) the disclosure threshold, \(\hat{q}\) solves the equation
\[
\hat{q} - c = \mathbb{E}[q|q < \hat{q}]
\]
Replacing \(\hat{q}\) with \(v_{nd} + c\) this becomes:
\[
v_{nd} = \mathbb{E}[q|q < v_{nd} + c]
\]

However, by equation (2) \(v_{nd}\) is the weighted average of \(\mathbb{E}[q|q < v_{nd} + c]\) and the smaller \(\mathbb{E}[q|q < v_{nd}]\), implying \(v_{nd} + c < \hat{q}\).

The remaining parts of this proposition are an immediate consequence of the previous parts and that expected firm value is decreasing in the disclosure probability of the high-cost firm.

Proof of Lemma 1

After disclosure in the first period a high-cost firm has an actual second period profit of \(q_2 - c\). It is useful to define \(\hat{q}\) as the second period profit minus the first period disclosure costs. This implies that a firm with high costs has a profit distribution of \(F^c = U[0,1]\) while a firm with no disclosure costs has a profit
distribution of $F^0 = U[c, 1 + c]$. This redefinition, does not change the fact that disclosure thresholds exist and are linear in $c$.

The equilibrium is still characterized by equation (2) with the additional complication that each type of firm uses a different profit distribution.

$$\frac{\mu F^0(v_{nd})}{\mu F^0(v_{nd}) + (1 - \mu) F^c(v_{nd} + c)} E^{F^0}(q|q < v_{nd}) + \frac{(1 - \mu) F^c(v_{nd} + c)}{\mu F^0(v_{nd}) + (1 - \mu) F^c(v_{nd} + c)} E^{F^c}(q|q < v_{nd} + c) = v_{nd}$$

There are four possible cases to consider depending on whether the disclosure decisions of each firm is certain or not.

$v \leq c$ Under this case the low-cost firm always discloses and if there is no disclosure the market knows that the firm is drawing from a distribution of $[0, 1]$. Thus we must have a threshold (for the real profits) of $\hat{q}^* - c = E[\hat{q} \hat{q} < \hat{q}^*] = \frac{c^*}{2}$. Thus there is an equilibrium where the low-cost firm always discloses, and the high-cost firms discloses real profits of $\hat{q} > 2c$ or actual profits of $q > 3c$.

$c < v \leq 1 - c$ In this case the equilibrium condition becomes:

$$\mu(v - c) \frac{v + c}{2} + (1 - \mu) \frac{(v + c)^2}{2} = v(1 - \mu) + (1 - \mu)v$$

$$v \in \{c, c(2\mu - 1)\}$$

The first solution is the one from the previous section. And the second one is invalid as $c(2\mu - 1) < c$.

$1 - c \leq v < 1$ In this case the high-cost firm never discloses and the equilibrium condition becomes:

$$\mu(v - c) \frac{v + c}{2} + (1 - \mu) \frac{1}{2} = v(1 - \mu) + \mu(v - c)$$

$$v = \frac{\mu - 1 + c\mu \pm \sqrt{(\mu - 1)(2c\mu - 1)}}{\mu$$

However, neither of these solutions fall in the range $[1 - c, 1]$ for $c \in (0, \frac{1}{4})$ and $\mu \in (0, 1)$.

$v > 1$ There is clearly no equilibrium in this case, as the best non-disclosing firm has no disclosure costs, and can thus separate itself from the other worse firms it is pooled with by disclosing.
Proof of Corollary 1
This corollary follows by plugging the uniform distribution into the indifference condition (2) and solving to get

\[ v_{nd} = c\sqrt{1 - \mu} + c \]

The expected probability of no disclosure under asymmetric information is

\[ c(1 - \mu + \sqrt{1 - \mu}) \]

While the probability of no disclosure for a firm with a known cost of \( c \) under the Uniform Distribution is \( 2c \). Thus the expected probability of no disclosure under asymmetric information is

\[ (1 - \mu)2c \]

As \( \mu \in (0, 1) \) we must have \( c(1 - \mu + \sqrt{1 - \mu}) > (1 - \mu)2c \) and there is less disclosure under asymmetric information.

Proof of Lemma 2
If both firms use a threshold of \( q_0 = q_c = c \) this lemma is vacuously true as \( c \) is the minimal level of profit.

It is also immediate that we cannot have \( q_c < q_0 \) as then a firm with high disclosure costs has a profitable deviation by not disclosing profits of \( q_c \). To see this is the case, note that as a low-cost firm finds it optimal to not disclose at \( q_c \) we must have

\[ q_c + V(0, c) + \delta V(0, 0) \leq q_{nd} + \mathbb{E}L_{nd}(\mu_{nd}) + \delta V(0, \mu_{nd}) \]

While for disclosure to be optimal for a firm with high disclosure costs we need

\[ q_c - c + V(0, c) + \delta V(0, c) \leq q_{nd} + \mathbb{E}L_{nd}(\mu_{nd}) + \delta V(0, \mu_{nd}) \]

For these two inequalities to hold we must have

\[ \delta V(0, c) - c \geq \delta V(0, 0) \]

which is a clear contradiction as \( V(0, 0) > V(0, c) \).

Proof of Proposition 2
For full disclosure to be an equilibrium it is enough to show that there is no
profitable deviation for a firm with a profit of \( c \). As the profit from deviating to non-disclosure is highest for a high-cost firm with \( q_1 = c \), the D.1 condition implies \( q_{nd} = c \) and \( \mu_{nd} = 0 \).

The equilibrium utility from disclosure is

\[
 c + \mathbb{E}L_d(\mu) + \delta \begin{cases} 
 V(1, 0) & \text{if } \hat{c} = 0 \\
 V(0, c) - c & \text{if } \hat{c} = c 
\end{cases}
\]

While the equilibrium utility from no disclosure is

\[
 c + \mathbb{E}L_{nd}(0) + \delta V(0, \hat{c})
\]

For a firm with no disclosure costs disclosure is optimal if

\[
 c + \mathbb{E}L_{nd}(0) + \delta V(0, 0) \leq c + L_d(\mu) + \delta V(1, 0) \\
\delta \leq \frac{2}{c}(2\mu(1 - c) - 1)
\]

While for a firm with high disclosure costs full disclosure is optimal if

\[
 c + \mathbb{E}L_{nd}(0) + \delta V(0, c) \leq c + L_d(\mu) + \delta (V(0, c) - c) \\
\delta \leq 2\mu(1 - c) - 1
\]

Unsurprisingly, the tighter constraint is for the high-cost firm. Thus we get a fully revealing equilibrium when

\[
\delta \leq 2\mu(1 - c) - 1
\]

If a fully revealing equilibrium exists, it is clearly the most informative equilibrium, implying it is the unique equilibrium satisfying condition D.1

**Proof of Proposition 3**

If a firm with no disclosure costs always discloses, then the utility from non-disclosure for a firm with high disclosure costs is

\[
 q_{nd} + \mathbb{E}L_{nd}(0) + \delta V(0, c) = \frac{c + q_c}{2} + (1 + \delta)V(0, c)
\]

While the utility from disclosing a profit of \( q \) is

\[
 q + \mathbb{E}L_d(\mu) + \delta V(\mu, c)
\]
Thus the threshold for disclosure for a firm with high disclose costs is

\[ q_c = c(3 + 2\delta - 4\mu(1 - c)) \]

For full disclosure to be an equilibrium strategy for a firm with no disclosure costs, the utility from revealing a profit of \( c \) and that \( \hat{c} = 0 \)

\[ c + \mathbb{E}L(1) + \delta V(1, 0) = c + (1 + \delta) \frac{1 + 2c}{2} \]

must be greater than the value of no disclosure

\[ q_n d + \mathbb{E}L_{nd}(0) + \delta V(0, 0) = \frac{c + q_c}{2} + V(0, c) + V(0, 0) \]

Solving for this inequality shows that disclosure of a profit of \( c \) is optimal if

\[ \delta \leq \frac{4(\mu(1 - c) - c)}{2 + c} \]

Uniqueness follows from the same argument used in Proposition 3.

For the second part of the proof remember that the probability of no disclosure in a static model is

\[ c(1 - \mu + \sqrt{1 - \mu}) \]

While the probability of no disclose in the dynamic model is

\[ (1 - \mu)(q_c - c) \]

Thus the probability of disclosure is higher in the dynamic model if

\[ (1 - \mu)(q_c - c) < c(1 - \mu + \sqrt{1 - \mu}) \]

\[ c \left( \frac{2(\mu - 1)(2(c - 1)\mu + \delta + 1) - \mu + \sqrt{1 - \mu} + 1}{2 + c} \right) > 0 \tag{5} \]

This probability is linear in \( \delta \) thus it is enough to check for the extreme levels of \( \delta \) which are \( \delta_1, \delta_2 \). For \( \delta = \delta_1 \) (5) becomes

\[ c \left( 1 - \mu + \sqrt{1 - \mu} \right) > 0 \]

While for \( \delta = \delta_2 \) (5) becomes

\[ c \left( \frac{2(\mu - 1)(2(c - 1)\mu - 3c + 2)}{c + 2} + 1 - \mu + \sqrt{1 - \mu} \right) > 0 \]

As the LHS is increasing in \( c \) it is enough to check for \( c = 0 \) for which the LHS equals zero. Thus as I have assumed \( c > 0 \) equation (5) is satisfied and there is
more disclosure in the dynamic model.

Proof of Proposition 4

A preliminary step before showing the main result is showing that when \( \delta > \delta_2 \) there is some disclosure by both firms. Clearly, there is always some disclosure by a firm with no disclosure costs. Assume to the contrary there is no disclosure by a firm with high disclosure costs.

The use of threshold strategies by both firms implies that:

\[
q_{nd} = \frac{q_0 + c}{2} + \frac{(\mu - 1)(q_c - c)(q_c - q_0)}{2(\mu q_0 + (1 - \mu)q_c - c)}
\]

\[L_{nd}(\mu_{nd}) = \mu_{nd} V(\mu, 0) + (1 - \mu_{nd})\left[\frac{q_0 - c}{q_c - c} V(\mu, c) + \frac{q_c - q_0}{q_c - c} V(0, c)\right]\]

Equation (ID0) gives that

\[
q_0 + L_d(1) + \delta(V(1, 0) - V(\mu, 0)) = q_{nd} + E L_{nd}(\mu_{nd}) \tag{6}
\]

For the firm with high disclose to find non-disclosure of \( q = 1 + c \) optimal we must have that

\[
1 + c + L_d(1) + \delta(V(0, c) - c) \leq q_{nd} + E L_{nd}(\mu_{nd}) + \delta V(0, c)
\]

Plugging in equation (6) and rearranging gives

\[
1 + c - \frac{1}{2} c \delta(c(\mu - 1) + 2) \equiv q_0^* \leq q_0
\]

However, evaluating equation (6) at \( q_0^* \) shows that disclosure is strictly preferred to non-disclosure at \( q_0^* \) and thus the disclosure threshold for a firm with no costs is less than \( q_0^* \) and not disclosing \( q = 1 + c \) is not optimal for a firm with high disclosure costs.

The proof of the main claim is in two parts. In the first part I show that when \( \mu \) converges to 1 (0) there is more (less) disclosure in the dynamic settings. I then show that the there is at most one value of \( \mu \) for which the static threshold is optimal in the dynamic case.

To prove the first part observe that as all the indifference conditions are continuous in all their arguments it is enough to show the desired result for a limiting \( \mu \) and then conclude via continuity that the same result holds for
extreme values of $\mu$.

The limit of the indifference conditions when $\mu \to 0$ is:

\[
q_c = c(3 + 2\delta)
\]
\[
q_0 = \frac{1}{2c}(2 + 4c + 2\delta + c\delta)
\]

Where as the limit of the disclosure threshold in the single period model is $2c$ and $3c$ respectively. $q_c$ is clearly greater than $3c$ and

\[
\frac{1}{2c}(2 + 4c + 2\delta + c\delta) > 2c
\]
\[
\delta > \frac{2 - 4c}{2 + c}
\]

Which is always true since $\delta > \delta_2$.

The limit of the indifference conditions when $\mu \to 1$ is:

\[
q_0 = c
\]
\[
q_c = c(1 + \delta)
\]

Where the limit of the disclosure threshold in the single period model is $c$ and $2c$ respectively. $q_c$ is clearly less than $2c$, while a firm with no disclosure costs uses a strategy of full revelation in both cases. However, the ratio of the derivative of the disclosure threshold WRT to $\mu$ in the static and the dynamic cases is $\infty$.\(^{17}\) Thus for high values of $\mu$ there is more disclosure in the dynamic setting.

Subtracting the two indifference conditions shows that $x = q_c - q_0$ equals

\[
x = \frac{c}{2}(c(\delta + 4)(\mu - 1) + 2\delta + 4(1 - \mu))
\]

(7)

Note that as $c < \frac{1}{4}$ this difference is strictly decreasing in $\mu$.

\(^{17}\)The derivative in the static case is calculated directly, and the derivative in the dynamic case is obtained via implicit derivation of the indifference conditions. L’hopital’s rule is used to calculate the ratio of disclosure thresholds at the limit, and then I evaluate the ratio at the solutions at $q_c = c$. 
is acting optimally and sets \( q_c = q_0 + x \). This equation becomes

\[
q_0 - \frac{1}{2}c^2 \delta(1 - \mu) = \frac{\mu (q_0 - c) (c + q_0)}{2(\mu (q_0 - c) + (1 - \mu) (q_0 + x - c))} + \frac{1}{2} (c + q_0 + x) \left( 1 - \frac{\mu (q_0 - c)}{\mu (q_0 - c) + (1 - \mu) (q_0 + x - c)} \right)
\]

\[
- c (1 - \mu) \left( \frac{(1 - c\sqrt{1 - \mu - c}) (q_0 - c)}{q_0 + x - c} + \frac{(1 - 2c)x}{q_0 + x - c} \right)
\]

Differentiating the RHS wrt \( x \) gives

\[
\frac{1}{2} (1 - \mu) \left( \frac{2c^3 \sqrt{1 - \mu}}{(-c + q_0 + x)^2} - \frac{2c^2 \sqrt{1 - \mu} q_0}{(-c + q_0 + x)^2} + \frac{2c^2 (q_0 - c)}{(-c + q_0 + x)^2} - \frac{x}{c - q_0 + (\mu - 1)x} + \frac{(c - q_0) (c - q_0 - x)}{(-c + q_0 - \mu x + x)^2} \right)
\]

Evaluating this derivative when \( q_0 \) equals that static threshold of \( c + c\sqrt{1 - \mu} \) gives

\[
\frac{1}{2} (1 - \mu) \left( \frac{2c^3 (\sqrt{1 - \mu} - (1 - \mu))}{(c\sqrt{1 - \mu} + x)^2} + \frac{c^2 (1 - \mu) + c\sqrt{1 - \mu} x}{(c\sqrt{1 - \mu} + x) (1 - \mu)} + \frac{x}{c\sqrt{1 - \mu} - \mu x + x} \right)
\]

Which is a positive expression.

Thus there is a single value of \( x \) for which the static threshold can be optimal in the dynamic setting. As \( x \) is decreasing in \( \mu \) this concludes the proof.

A similar argument shows that there is also a single value of \( x \) for which a firm with high disclosure costs uses its static threshold in a dynamic setting.

**Proof of Proposition 5**

Re-writing the indifference conditions gives

\[
q_0 = q_{nd} + \delta(V(\mu, 0) - V(1, 0))q_c - c = q_{nd} + \delta(V(0, c) - V(\mu, c))
\]

As \( 1 > \mu > 0 \) we have that \( V(\mu, 0) > V(1, 0) \) and \( V(0, c) > V(\mu, c) \) thus the indifference conditions are the indifference conditions for the static case with an additional positive term associated with non-disclosure. As \( q_{nd} \) is increasing in \( q_c \) the direct incentive for each manager to avoid disclosure is augmented by the increase in \( q_{nd} \) due to less disclosure by the other type of manager, and dynamic considerations decrease voluntary disclosure.
Appendix B - Uniqueness of equilibrium in the second period

Apriori, equation (2) might have multiple solutions, which would allow for multiple equilibria in the second period of the dynamic model. Such multiplicity would complicate the analysis since different continuation equilibria can be selected in the second period depending on the outcome of the first period. In order to avoid this complication, I offer a simple condition that ensures a unique continuation equilibrium.\footnote{This condition is satisfied by many common distributions such as the Uniform Distribution, the Exponential Distribution, and the Log-Normal Distribution (for sufficiently large costs). Furthermore, the Normal Distribution (that does not satisfy the assumption of positive support) also satisfies this condition and permits a unique equilibrium.}

**Condition A.** \( c \leq \frac{G(c)}{g(c)} \) and \( \frac{G(q)}{g(q)} \) is non decreasing in \([c, \bar{q}]\).

**Lemma 3.** If \( G(q) \) satisfies condition (A) there is a unique solution to equation (2).

**Proof.** The indifference condition (2) can be rewritten as:

\[
0 = \int_0^{v_{nd}} qg(q)dq - \mu v_{nd}G(V_{nd}) + (1 - \mu)\left( \int_{v_{nd}}^{v_{nd}+c} qg(q)dq - v_{nd}G(v_{nd}+c) \right)
\]

The derivative of the RHS w.r.t. to \( v_{nd} \) is:

\[
 cg(v_{nd} + c) - G(v_{nd} + c) - \frac{\mu}{1 - \mu}G(v_{nd})
\]

In order for this derivative to be negative for all values of \( \mu \), we must have

\[
 cg(v_{nd} + c) < G(V_{nd} + c) \iff c < \frac{G(v_{nd} + c)}{g(v_{nd} + c)}
\]

As \( v_{nd} \geq 0 \), assuming the inverse hazard ratio is non decreasing and \( c \leq \frac{G(c)}{g(c)} \) implies this derivative is negative and that there is a unique \( v_{nd} \) that solves equation (2)
References


