Outsourcing patent enforcement: The effect of “Patent Privateers” on litigation and R&D investments.

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Abstract

Monetizing patent portfolios has become a major part of business strategy. The rise of firms specialized in acquiring and monetizing patents has led to an important policy debate regarding their impact on innovation incentives and litigation. We present a model of innovation, licensing, and litigation, to study the effect of Patent Assertion Entities (PAEs) on ex-ante R&D incentives and litigation. We identify two effects of outsourcing patent monetization to PAEs. First, it leads to larger litigation threats in equilibrium, and lowers the aggregate industry profits of firms that carry out R&D. This rent extraction effect has a negative impact on incentives to invest in R&D. Second, PAEs increment patent monetization, which increases the payoff of the ‘winner’ of a multi-object patent race and decreases the payoff of the ‘loser’. This winner premium effect intensifies the incentives to invest in R&D. Our model shows that it is possible for PAEs to increase incentive to invest in R&D, although they do not invest in R&D, they do not produce anything, and they extract positive rents from operating companies.

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1 Introduction

The U.S. patent system has changed over the past few years. Firms today have new ways of monetizing their patents: patent assertion entities (PAE), also known as non-practising entities (NPE) or “patent trolls,” have risen to prominence by buying up significant numbers of patents and bringing alleged infringers to court, and by using the threat of litigation to extract license payments. This recent development has lead to much public and academic debate on the merits of PAEs and on the effect that they have on innovation and litigation. There is no doubt that PAE activities significantly impact the way the patent system works. This can be seen clearly from the multitude of bills that have been passed or proposed in Congress\footnote{These include for example the SHIELD Act, the Patent Quality Improvement Act, America Invents Act, and the End Anonymous Patents Act} and the President’s public stance on the issue\footnote{See “Patent Assertion and U.S. Innovation,” published in June 2013 by the Executive Office of the President and prepared by the CEA, NEC & OSTP.}

In this paper we present a theoretical model of innovation, licensing, and litigation, in order to study the effect of PAEs on R&D incentives of competing firms. The model incorporates many of the key features of the patent system today: first, litigation is costly; second, we allow for probabilistic patents, as in Lemley and Shapiro (2005); third, we focus on complex products which use multiple patentable pieces of technology. This set of features is especially relevant to high tech industries, where PAEs have been most active.

We find that PAE activity leads to larger litigation threats in equilibrium, that PAEs weakly lower the aggregate industry profits of firms which carry out R&D, and that, perhaps surprisingly, in some cases they change industry payoffs in a way that increases the incentives to invest in R&D. When a firm decides how much to invest in R&D, its investment has two effects: it decreases the expected time to discover all technologies that are necessary for production, thus allowing all operating firms to capture the rewards from commercialization sooner, and it also makes it more likely that the firm will have a relatively larger share of the patents covering the necessary technologies, thus allowing it to capture a larger share of industry profits. The intuition behind our result is fairly simple: PAEs affect R&D incentives through two separate channels; first, they extract a part of overall industry profits in rents (we call this the rent extraction effect); second, PAEs change the rewards to innovation in a way that increases the payoffs of the “winner” of a patent race and decreases the payoffs of the “loser” of a patent race (we call this the winner premium effect). The first effect weakens the incentive of a firm to invest in R&D, because the reward to selling a product is smaller, so
there is less of an incentive to obtain it earlier. The second effect, on the other hand, enhances
the firm’s incentive for R&D—because PAE activity increases the payoff to the “winner” of a
patent race and decreases the payoff to the “loser,” the incremental payoff from each patent is
therefore larger and thus the incentive for R&D is stronger. Overall, under some stability con-
ditions, we find that PAEs enhance the incentive for innovation if the latter effect dominates
the former, and vice versa.

In the absence of PAEs, many competitors with similarly-sized patent portfolios will often
engage in a tacit “IP truce,” whereby neither firm is willing to sue its rivals for infringing
on its patents, because the rivals’ own portfolios act as a deterrent. Since going to court is
costly for both parties, even if one firm has a somewhat larger portfolio than its rival (i.e.
expects a positive overall payoff if it triggers a “patent war”), the net benefit from enforcing
its patents (after accounting for the cost of a potential counter-suit) may be less than the
expected legal costs. In such cases even a firm with a stronger patent portfolio may not have
a credible litigation threat. Hence some of the value of the firm’s patents is lost, because
the legal transaction costs make it impossible to enforce those patents. Countersuing plays
a crucial role on litigation strategy when firms can use their patents as defensive weapons.
Some salient examples of countersuing are Apple vs HTC, where HTC countersued with 2
patents, or Yahoo vs Facebook, where Facebook countersued with 10 patents.

Because a PAE cannot be countersued, its litigation incentives are stronger than those of
an operating firm, conditional on having the same patents. The firm with a stronger patent
portfolio can benefit from the PAE by allowing it to enforce some of the patents which would
otherwise not be monetized due to legal transaction costs and the threat of countersuing. If in
equilibrium, the PAE extracts additional rents from the rival firm, this excess must somehow
be divided among the original owner of the patent and the PAE that monetizes it. Hence,
the original inventor’s payoff must weakly increase, and the PAE provides higher incentives
to obtain more patents. In fact, when the PAE allows more patent monetization there are
two effects: 1) the firm with the weaker portfolio loses more compared to the tacit “IP truce”
equilibrium in absense of PAEs; 2) the firm with the larger portfolio can capture some of
the extra surplus generated by the PAE, determined by its bargaining power, while the rest
goes to the PAE as rents. Notice that both of these effects push the incentives for patenting
in the same direction: they both make being the firm with a larger portfolio more profitable.

By enhancing patent monetization, the PAE has the ability to weakly increase the payoff of a
firm with a larger patent portfolio, at the expense of charging higher license fees to a firm with
smaller portfolio. In our model, the PAE bilaterally bargains with the operating firms from
which acquired its patent portfolio, and in some cases the PAE will extract positive rents from
the industry. Notice that the PAE does not invest in R&D, nor is using the acquire patents
to produce, but even then it is able to generate positive rents. The rents extracted by the
PAE lower the expected industry profits which implies that, from an ex-ante perspective, the
presence of the PAE provide less incentives for firms to discover sooner all the technologies
necessary to create the final product.

So when firms endogenously decide how much to invest in R&D, they foresee a larger reward
to the discoveries that this R&D will generate, and thus have a stronger incentive to invest.
But also the PAE lowers total industry profits, which provide less incentives to invest more
in order to start selling the final product sooner. One of the contributions of this paper is to
show that even when PAEs extract positive rents from the market, they can still intensify the
R&D investment.

To evaluate the effect of PAEs from a welfare perspective, we characterize conditions under
which the firms under-invest and over-invest in equilibrium (in the absence of PAEs) relative
to the social planner’s first- and second-best outcomes. The model provides some comparative
statics. For example, the firm R&D equilibrium features under-investment when firms are less
patient, or when consumer surplus is large, or when the final product is more complex (i.e.
involves more pieces of technology), or when patent protection is weak. Most interestingly,
we show that when firms under-invest in innovation, PAEs may be beneficial and increase
equilibrium investments, pushing them closer to the social optimum. This happens precisely
when the winner premium effect dominates the rent extraction effect. Conversely, when firms
under-invest in R&D, PAE activity will be harmful for innovation if rent extraction dominates
the winner premium.

One of the key features of our model is that all innovation is endogenous: firms decide how
much to invest in R&D in anticipation of the rewards to patenting, which can come in the
form of product sales, patent trade, licensing, or litigation revenue. The goal of the patent
system is to provide incentives to invest in R&D and generate innovation, so this is the most
appropriate context in which to evaluate the effect of PAEs. This is in contrast to most
existing papers on the subject of PAEs, which take R&D investments as exogenous and look
at litigation and licensing incentives in a fixed patent landscape (see for example Bessen and
Meurer (2014), who attempt to quantify the legal costs of PAE litigation; Choi and Gerlach

\[3\] This is similar to what we would expect to happen in a hypothetical world with zero legal transaction
costs (and without PAEs), because in that case each patent could be credibly enforced in court, which would
be reflected in the price of a license, and so firms would anticipate larger rewards to R&D.
In practice, PAEs differ significantly in their business strategies and, as mentioned in Risch (2012), there is still much research to be done to have a clear picture of the PAE business. One relevant aspect of the PAE business is the source of the patents they own. In recent years they have included universities (including deals whereby a PAE buys the rights to future patents), individual inventors, companies which have at some point invested in R&D and produced commercial products, but are looking to liquidate their portfolios (e.g. due to bankruptcy, or because the company is failing to monetize the value of its patents), as well as actively producing firms. The latter source of patents is most relevant to our model, because we assume that every firm that invests in R&D has the capacity to enter the final product market (as opposed to universities and individual inventors). Fischer and Henkel (2012) studies the characteristics patent acquired by PAEs, in contrast of those acquired by operating companies. They find that PAEs acquire a large number of patents from producing firms with more than 100 employees.

In this paper, we focus our attention on one particular PAE business strategy called *Patent Privateering*, which describe PAEs that acquire patents from operating companies to file infringement lawsuits against the sellers’ competitors. In some cases, the producing company arguably sets up a PAE precisely for the purpose of extracting revenue from its competitors through the threat of litigation, which would not be credible otherwise due the size of the competitors’ own patent portfolios. PAEs that are set up in this way are often surrounded by secrecy, because operating firms may have reputational concerns and because they generally do not want to be seen as being pro-litigation. In our model, we abstract away from this particular issue and assume that the PAE is an independent entity not directly controlled by any of the producing firms.

When firms are deciding on what to do with their intellectual property assets, they can attempt to monetize them on their own, or they can sell them to a buyer that offers a better deal. Firms holding large patent portfolios as result of their R&D investments may increase the returns on research by liquidating some of their patents. Examples of producing firms that have sold significant numbers of patents to PAEs include Alcatel-Lucent, British Telecom, Digimarc, Ericsson, Kodak, Micron Technology, Microsoft, Nokia, Sony, and others. Micron

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4They are called spanning PAEs in Scotto Morton and Shapiro (2014).
Technology, for example, is a multinational corporation based in Idaho, which has recently been named one of Thomson Reuters’s top 100 global innovators, and is one of the largest memory chips makers in the world (it owns the Crucial memory brand, among others). In 2005, Micron’s patent portfolio was just over half the size of IBM’s, and during the same year it invested about $600 million in R&D. Since then, it has sold significant numbers of patents (at least 20% of its portfolio) to Round Rock, a patent assertion entity, in multiple transactions in 2009 and 2013. Round Rock has subsequently used those patents in litigation against multiple large companies that used products provided by Motorola, Smartrac and others, which eventually settled for licenses.

Other examples of operating companies selling patents to a PAE include Nokia and Sony, which sold some of their portfolios to MobileMedia, a PAE which subsequently sued Apple, HTC and Research In Motion (in the case of Apple, which was the first to go to trial, a jury has already found that some of their products infringed on several of MobileMedia’s patents). Another example can be found at Intellectual Ventures, owner of arguably one of the largest patent portfolios in the world, which owns patents acquired from manufacturing entities such as Bell Atlantic, Image Inc., and Kodak. Further examples of important patent acquisitions by a PAE from an operating company include MOSAID, which acquired patents from Microsoft and Nokia in 2011, IPvalue and British Telecom, and Unwired Planet and Ericsson. Although most of the action occurs in the high-tech industry, the patent privateering phenomena is also present in other industries. For example, in 2006 Nike sold part of its patent portfolio to Cushion Technologies, LLC, that later on sued several of Nike’s rivals in the running shoe market.

An important feature that we capture in our model is the complexity (or modularity) of products. Nowadays, products are protected by many different patents, as each one of its components is individually patented. For instance, Apple holds more than 1,200 patents protecting the iPhone including software, hardware, and design patents. Lloyd et al. (2011) shows evidence of the amount of patents involved in the legal protection of one product.

Patent Privateering is an important phenomena and needs to be studied in more detail. On

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April 5, 2013, Google, BlackBerry, EarthLink, and Red Hat, sent a letter to the Federal Trade Commission and the Department of Justice to ask for more scrutiny specifically on patent privateering. An extract of this letter makes clear the importance of this patent monetization strategy:

“PAEs impose tremendous costs on innovative industries. These costs are exacerbated by the evolving practice of operating companies employing PAE privateers as competitive weapons. The consequences of this marriage on innovation are alarming. Operating company transfers to PAEs create incentives that undermine patent peace.[...] We therefore urge the antitrust agencies to study carefully the issue of operating company patent transfers to PAEs”

In the next section we review some of the large literature on PAEs, and some recent papers addressing somehow related research questions. In section 3 we introduce a model of R&D, licensing, and litigation. In section 4 to 5, we solve the licensing and litigation game and present our main results. In section 6 we solve the endogenous R&D decision. In section 7 discusses the welfare implications of our results and, finally, in section 8 we summarize our findings and offer some policy implications.

2 Literature Review

Khan (2005) shows that commercialization of patents is not a phenomena particularly tied to high technology products. Companies whose sole business is the monetization of patents have existed for a long time, called in the past “patent sharks” rather than “patent trolls”. Magliocca (2006) describes “patent sharks” as entities that extracted money from innocent individual farmers and railroad companies. One famous illustration is the independent inventor Jerome Lemelson, who signed licenses with many companies for the use of his patents. In recent years, the proliferation of companies focused on the assertion, rather than the commercialization, of patents has open an important debate. Chien (2010) studies this proliferation of PAEs, and rise of strategic management of patents. In particular, she emphasizes the importance of holding large portfolios to sustain “patent peace” among operating companies through the threat of countersuing.

Empirical research on PAEs is scarse, mostly due to the difficulty of finding the data. The majority of the empirical studies have been restricted to a subset of firms for which data is available, and some arguments against PAEs have been based on anecdotical evidence or
isolated cases. [Risch (2012) and Fischer and Henkel (2012)] have tried to shed light on the practices of PAEs by analyzing the patent portfolios of a sample of firms. An important finding of these papers is that, in their sample, PAEs acquire patents of relatively good quality (in terms of validity), which goes against the commonly held belief that PAEs try to enforce bad quality patents. [Shrestha (2010)] finds similar results, when compares the forward citations of patents acquired by PAEs versus those acquired by operating companies. An important empirical finding is that PAE do not acquire all its patents from individual inventors. [Fischer and Henkel (2012)] find that about 65% of the patents acquired by PAE came from operating companies with more than 100 employees.

[Bessen et al. (2011)] estimates the cost imposed by PAEs to operating companies. Analyzing stock market events around NPE lawsuit filings, they find a loss of about half a trillion dollars to defendants over the period 1990-2010. [Bessen and Meurer (2014)] estimate that the direct costs of PAE assertions (not including diversion of resources, delays in new products, and loss of market share) was about $29 billion in 2011. These studies, highly cited in the media by the large amount of rents extracted by PAEs, are not without caveats and critiques to their methodology. [Risch (2014) and Cotropia et al. (2013)], for example, questions the definition of PAE used in these papers to estimate the costs. In particular, [Cotropia et al. (2013)] provides a finer classification of PAEs by different types (universities, individual inventor, IP holding companies, etc) for all patent litigation cases in the years 2010 and 2012. PAE business strategies vary a lot, and different companies have found different ways of monetizing patent portfolios. There is no “one-size-fits-all” definition for PAE strategies.

[Scott Morton and Shapiro (2014)] provides a description of the different strategies employed by PAEs to monetize their patents. They also provide a simple model of PAE intermediation between an individual inventor and an operating company. In their baseline case, the individual inventor cannot monetize its patent, and the operating company infringes on the patent to produce the final product. The PAE acquires the patent from the individual inventor and it has the ability to enforce it. In their model, when the PAE does not transfer enough rents to the original inventor, PAEs will have a negative impact on welfare. The main different between their model and ours is the origin of the patents.

[Cohen et al. (2014)] present a model of PAE formation. In their model, initially two firms trying to enter the market and each firm owns one invention of exogenously given quality. If one firms has very low quality, it will stay out of the market and it will act as a patent troll. In our model, we endogenize the quality of the innovation, and also allow for entry decisions. [Cosandier et al. (2014)] studies defensive patent acquisition services, strategy that is utilized
by, for example, the company RPX Corporation. Finally, [Tucker (2014)] studies the effect of patent litigation on VC investments and concludes that frequent litigators are associated with a direct and negative effect on VC investments.

3 Model Overview

Two firms (A and B) race to discover $N$ pieces of technology, which we call *components*, in order to produce and sell a final product that incorporates all of them. The timing of the model is as follows: first, firms invest in R&D and patent their discoveries; second, observing the realization of patent portfolios after the R&D stage, firms have the option of buy or sell patents; third firms decide whether to enter the final product market; fourth, firms engage in patent licensing in the shadow of litigation; fifth, if a firm has entered the final product market without patents or licenses on all $N$ components, it can potentially be sued for patent infringement. PAEs interact with producing firms after they enter the market.

<table>
<thead>
<tr>
<th>R&amp;D investments</th>
<th>Patent trade</th>
<th>Entry</th>
<th>PAEs</th>
<th>Licensing</th>
<th>Litigation</th>
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**Figure 1:** Timing of the events in the model.

Firms simultaneously make sunk R&D investments to discover the $N$ components, and the discovery of a particular component arrives stochastically, given the R&D investments. We assume as in Loury (1979) that R&D investment is a one-time fixed investment, rather than a flow investment that can be revised upon the realization of uncertainty as in Lee and Wilde (1980). The cost of investing $z$ units of R&D for the firms is $c_I(z)$, where $c_I(\cdot)$ is increasing, convex, differentiable and $c_I(0) = 0$. Fixing the firm’s R&D investments $x$ and $y$, respectively for firm A and B, firm A is the first to discover any one particular component independently with probability $p(x, y) = \frac{h(x)}{h(x) + h(y)}$, where $h(\cdot)$ is increasing, concave, differentiable and $h(0) = 0$. This function is derived from independent exponential arrivals. If, for a given level of R&D $z$, each component arrives independently at time $\tau_c(z) \sim \exp(h(z))$, then firm A discovers a component first if $\tau_c(x) < \tau_c(y)$ which occurs with probability $\frac{h(x)}{h(x) + h(y)}$. Hence, the number of patented components for a particular firm follows a binomial distribution, and the probability that firm A discovers exactly $k$ components is given by

$$P(k; x, y) = \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k}.$$
Discoveries are publicly observable, and the firm which discovers a component immediately and costlessly obtains a patent on it. At the end of the R&D stage the patent portfolio of each firm is fixed. The expected time to complete the R&D stage is endogenously determined by the level of investment of the firms. The time at which a particular component \( i \in \{1, \ldots, N \} \) is discovered is given by \( \tau_i(x, y) = \min\{\tau_c(x), \tau_c(y)\} \). Production can take place only when every component has been discovered by some firm, since firms must either invent and patent, or obtain a license for, or imitate each of them, to be able to produce. The time at which firms will enter the market and produce is therefore given by \( \tau(x, y) = \max_{i=1, \ldots, N} \{\tau_i(x, y)\} \), which is distributed according to \( F(\tau; x, y) \).

Once patent portfolios are determined, firms can engage in patent trade. We assume that the original inventor of a patent always retains a license for his invention, even after assigning the patent to a new firm. In consequence, the original assignor of a patent cannot infringe on that patent, even after it does not longer owns it.

Given the patent portfolios after the patent trade, firms simultaneously decide whether to enter the final product market. Entry is not blocked by the lack of patents or licenses for some components, since firms can freely and immediately imitate any component discovered by any other firm. Industry profits (i.e. before accounting for license or litigation costs) are modeled in reduced form: if both firms enter the market, each one of them makes profit \( \pi > 0 \) by selling the final product. If only one firm enters, it monopolizes the market and obtains \( \pi_m \) in the final product market.

Next, firms engage in patent licensing with any competitor that has entered the final product market, and licenses are determined under the threat of litigation. We assume that license prices are set through Nash bargaining over the surplus that is generated by a licensing deal, relative to the firms’ outside options of not licensing and potentially going to court. If PAEs are available, firms can decide to trade patents with the PAE after entry and before they license with their rivals.

Finally, if a firm decided to enter the product market without licenses or patent protection for all the \( N \) components, it may be sued for infringement, and the court will decide whether the final product infringes on unlicensed components. We assume that patents are valid

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10 I.e., for simplicity we assume away the possibility of trade secrets or strategic delay in patenting.

11 Notice that firms have the same market size, despite having potentially asymmetric patent portfolios.

12 See Spier (2007) for a discussion of the role of bargaining power in settlement outcomes. In a separate working paper, Lemus and Tenyakov (2013) study how bargaining power and injunctions affects the incentives for litigation in the presence and absence of a PAE.
(“strong”), but probabilistic: a firm’s product infringes on each patent with probability $\beta > 0$, which is independent across patents. We further assume that going to court is costly: each side must pay $c > 0$ in legal fees per lawsuit, and that the defendant may bring counter-claims (defensive countersuing) to the court at no additional cost. If a firm is found to infringe on a patent, it must pay the patent-owner a per-patent infringement fee $R > 0$.

In the next section we proceed to solve the model by backward induction. We derive the continuation payoffs from the last stage first in an economy without PAEs and then we re-derive the continuation payoffs with PAEs. Then, we examine the R&D decisions when firms forsee the continuation payoffs derived before.

4 Patent trade, entry, licensing and litigation – Without PAEs

We solve the game by backward induction, assuming first that PAEs do not exist. We start by solving the licensing and litigation stage, and we move backwards to entry and patent trade.

4.1 Licensing & Litigation

We analyze the licensing and litigation stages, for any given distribution of patents, after one or both firms entered the final product market.

Licensing and litigation payoffs when both firms enter

Consider the situation where firms A and B enter the market for the final product, which is composed of $N$ components, and the R&D investments determined that firm A patented $n$ components, while firm B patented $m$, with $n + m = N$. When $0 < n < N$ both firms have entered the product market with incomplete patent protection. When firm A sues firm B using its complete patent portfolio, given that each infringement claim is evaluated independently, the probability that firm B’s product infringes on exactly $k$ out of the $n$ patents owned by

$^{13}$We assume that $c$ is independent of the number of patents being litigated.

$^{14}$Because we model industry profits in reduced form, we also focus on per-patent royalties, rather than per-sale royalties. This avoids the complication of how royalties themselves affect pricing, which is not central to this paper.
firm A is given by
\[ \binom{n}{k} \beta^k (1 - \beta)^{n-k}, \]
and the expected payment in royalties received by A is given by
\[ \sum_{k=0}^{n} \binom{n}{k} (Rk)\beta^k (1 - \beta)^{n-k} = Rn\beta. \]

Because countersuing is free, firm B will counter-sue using its entire portfolio to obtain expected royalties of \( Rm\beta \). Thus, firm A’s expected payoff from going to litigation is \( R\beta(n - m) - c \), and B’s expected payoff is \( R\beta(m - n) - c \). In this situation, should licensing negotiations fail, firm A is willing to initiate litigation if and only if \( R\beta(n - m) > c \), while firm B is willing to initiate litigation if and only if \( R\beta(m - n) > c \) (i.e., these are the cases where either side has a positive-expected-value suit, as discussed for example in Shavell (1982) and Nalebuff (1987)).

If one firm has a credible threat of litigation, firms will bargain to avoid losing a total of \( 2c \) in joint surplus due to litigation costs. For simplicity, we assume equal bargaining power in the Nash bargaining solution at this stage. Denoting by \( \hat{c} = \frac{c}{R\beta} \) and \( V = R\beta \) and defining the function
\[ T(n, m) = \begin{cases} V \cdot (n - m) & \text{if } |n - m| > \hat{c} \\ 0 & \text{if } |n - m| \leq \hat{c} \end{cases}, \]

We have three relevant cases to analyze:

1. If firm A has a credible litigation threat: \( n - m > \hat{c} \), firms cross license and firm A receives the transfer \( T(n, m) = V \cdot (n - m) - c + \frac{1}{2}(2c) = V \cdot (n - m) \).
2. If firm B has a credible litigation threat: \( m - n \geq \hat{c} \), firms cross license and firm B receives the transfer \( T(m, n) = V \cdot (m - n) - c + \frac{1}{2}(2c) = V \cdot (m - n) \).
3. If no firm has a credible litigation threat: \( -\hat{c} < n - m < \hat{c} \), and \( T(n, m) = 0 \).

This means that for fixed portfolio sizes \((n, m)\) either one of following two cases occur: one of the firms has a relatively large enough portfolio, so the litigation threat is credible and that firm receives a payment from cross-licensing portfolios; or firms have portfolios of similar sizes, so litigation is not credible and firms produce while tacitly agreeing to not go to court. The figure below depicts the function \( T(n, m) \) for all possible combination of patent portfolios.
No licensing

Firm A pays

\[ V(m - n). \]

Firm A gets

\[ V(n - m). \]

Figure 2: Licensing transfers are shown for different portfolio configurations. Cross licensing agreements featuring positive transfers occur only when \( |n - m| > \hat{c} \).

We denote by \( U^A_{E,E}(n,m) \) the payoff of firm A after both firms entered the product market and bargained over their patent portfolios, which sizes are \( n \) and \( m \), for firm A and for firm B, respectively. By definition, \( U^A_{E,E}(n,m) = \pi + T(n,m) \), or equivalently:

\[
U^A_{E,E}(n,m) = \begin{cases} 
\pi + V \cdot (n - m) & \text{if } n - m > \hat{c}, \\
\pi & \text{if } |n - m| \leq \hat{c}, \\
\pi - V \cdot (m - n) & \text{if } m - n > \hat{c}
\end{cases}
\]

By symmetry, \( U^B_{E,E}(m,n) = U^A_{E,E}(n,m) \).

Licensing and litigation when only one firm enters

Consider now the case where only one firm enters the product market, say firm A, and A has \( n \) patents, and B has \( m \) patents. Because B is not producing anything, firm A’s portfolio is worthless in litigation. Firm B’s portfolio, however, can be monetized as long as firm B is willing to litigate, that is \( m > \hat{c} \). In this case, the negotiated license fees are given by \( T(m,0) = Vm \), as long as \( m > \hat{c} \). Notice that in this case, firm B is actually operating as a PAE, although it invested in R&D.

If \( m \leq \hat{c} \), firm B cannot monetize its portfolio and firm A produces as a monopolist without any credible threat of litigation. The total payoffs when only firm A enters the product...
market, denoted by $U_{A,NE}(n,m)$ and $U_{B,NE}(n,m)$, are given by

$$U_{A,NE}(n,m) = \begin{cases} \pi_m & \text{if } m \leq \hat{c}, \\ \pi_m - Vm & \text{if } m > \hat{c} \end{cases}, \quad U_{B,NE}(n,m) = \begin{cases} 0 & \text{if } m \leq \hat{c}, \\ Vm & \text{if } m > \hat{c}. \end{cases}$$

### 4.2 Entry decisions

We now analyze the optimal product market entry strategies, given the continuation payoffs described above, for a fixed patent portfolio, where firm A has $n$ patents and firm B has $m$ patents, and $n + m = N$. Since the problem is symmetric for both firms, we focus on firm B’s optimal entry decision, taking the decision of firm A as given. We assume that duopoly profits are larger than the cost of litigation, i.e. $\pi > c$, and that monopoly profits are larger than twice duopoly profits, i.e. $\pi_m > 2\pi$. We use the notation $\hat{\pi} \equiv \frac{\pi}{V}$.

**Lemma 1.** When $\hat{\pi} > N$ it is a dominant strategy for each firm to enter the final product market.

**Proof.** 1. Suppose firm A enters.

   (a) If $m \leq \hat{c}$ and firm B does not enter, firm B gets zero because the monetization of its portfolio is not profitable. If firm B enters, then it gets $\pi + T(m,n)$ which is non-negative if and only if $\hat{\pi} > n - m$. Since $n - m < N$ and by assumption $\hat{\pi} > N$, it is optimal for firm B to enter.

   (b) If $m > \hat{c}$ firm B’s outside option is to stay out of the market and use its portfolio to obtain licenses from firm A. Therefore,

   i. Firm B enters when $m - n > \hat{c}$ iff $\pi + V(m - n) \geq Vm$, that is $\hat{\pi} \geq n$

   ii. Firm B enters when $|m - n| < \hat{c}$ iff $\pi \geq Vm$, that is $\hat{\pi} \geq m$

   iii. Firm B enters when $n - m > \hat{c}$ iff $\pi - V(n - m) \geq Vm$, that is $\hat{\pi} \geq n$

   Under the assumption $\hat{\pi} > N$, firm B optimally enters in each one of the cases above.

2. Suppose firm A stays out.

   (a) If $n \leq \hat{c}$ firm A cannot monetize its patents, and it poses no credible litigation threat to firm B. In this case, firm B always enters and monopolizes the market.
(b) If \( n > \hat{c} \) firm A has a credible litigation threat and firm B needs to trade off the risk of litigation versus the gains from monopolizing the market. Therefore, firm B will enter as long as \( \pi_m - V_n \geq 0 \), i.e. \( \hat{\pi}_m \geq n \). Notice that firm A is not producing anything in this case, and therefore firm B’s portfolio cannot be used to accuse firm A of infringement.

As we noted above the condition \( \hat{\pi} > N \) implies that regardless of firm A’s entry decision, firm B always enters the market. Since the problem is symmetric from firm A’s perspective, it is a dominant strategy for firm A to enter as well, and the unique optimal outcome in the entry stage is for both firms to enter.

4.3 Patent Trade

In this section we analyze the possibility of patent trade among firms A and B after the R&D stage is over. We again consider the portfolios \( n \) an \( m \) for firms A and B, respectively. We assume that an original patent assignor always retains a license, even after selling the patent to another party. That is, the first firm to discover a component will always have protection over it. We also assume that firms cannot sign a contract that only allows one firm to enter and monopolize the market, even when this might be profitable to do. In other words, we assume that the anti trust authorities will be vigilant and prevent firms from signing these type of contracts.

Under these considerations firms are indifferent between trading patents of buying licenses in the following stage. Suppose firm B decides to sell \( \ell \leq m \) patents to firm A at some price \( p(\ell) > 0 \). In our model, selling the patents but keeping a license it is equivalent to selling a license. This is because the buyer cannot sue the seller on those patents, and there are only two players in the game. Under our assumption \( \hat{\pi} > N \), both firms always find profitable to enter the final product market regardless their patent portfolio size. Therefore, the price of these patents is given by \( p(\ell) = T(n, m - \ell) - T(n, m) \), which is the difference between the licensing fee charged by firm A when firm B can only use \( m - \ell \) of its patents, and the license fee when firm B can use all its portfolio.
### 4.4 Continuation payoffs without PAEs

In this section we summarize the subgame equilibrium: Firms do not trade patents, they both enter and they bargain over its portfolios. When firm A discovered and patented $n$ components, and firm B the remaining $m$ components, the continuation payoffs are $U^A(n, m) = \pi + T(n, m)$ for firm A, and $U^B(n, m) = \pi - T(n, m)$ for firm B. Since $n + m = N$ we can write $U^A(n, N - n) = \pi + T(n, N - n)$. Notice that $|n - m| > \hat{c}$ is equivalent to $|2n - N| > \hat{c}$, and in that case the $T(n, N - n) = (2n - N)V$. Firm A’s continuation payoff as a function of $n$ is depicted below.

**Figure 3:** Continuation payoff without PAEs after the R&D stage for firm A, when it discovered $n$ components, while its rival discovered $m = N - n$.

### 5 Patent trade, entry, licensing and litigation – With PAEs

In this section we introduce a PAE into the model, and analyze the continuation payoffs after the R&D stage. The PAE is strategically different from producing firms since it cannot be counter-sued, by definition. Therefore, producing firms have no tools to defend themselves against a PAE. The only risk that PAEs face in litigation is the randomness of court decisions. In our model the PAE begins the game with no patents, since it does not invest in R&D. The only way for the PAE to acquire patents is to buy them from firms that invested in R&D, once the research stage is over, and after the entry decisions have been made. When the PAE
acquires \( n' \) patents from a producing firm, that firm is granted a license for the patents it sold, so the PAE cannot sue the original inventor. A producing firm leaves itself more vulnerable to litigation after selling some of its patents, since it cannot use them defensively; but it may also generate additional revenue from the price that the PAE would be willing to pay for the patents, since the PAE can use those patents offensively without the countersuing threat. 

Suppose a PAE has acquired \( n' > \hat{c} \) patents from firm A and is planning to sue firm B. The expected payoff from litigation for the PAE is \( V \cdot n' - c \), and for firm B is \(-V \cdot n' - c\). Here we also assume symmetric bargaining power in the negotiation of licenses between the PAE and a producing firm. When the litigation threat is credible \((n' > \hat{c})\), firms will bargain over the surplus of avoiding litigation. Firm B is willing to pay the PAE

\[
T(n', 0) = V \cdot n' - c + \frac{1}{2}(2c) = Vn'
\]
to avoid litigation.

We adopt the following notation: \( n \) are the number of patents originally invented by firm A, \( n' \) are the number of patents sold by firm A to the PAE. Analogously, \( m \) are the number of patents originally invented by firm B, \( m' \) are the number of patents sold by firm B to the PAE. The values \( n' \) and \( m' \) will be endogenously determined in equilibrium, and we present its full characterization in section 5.1.

### Licensing and litigation payoffs when both firms enter

Consider a fixed allocation of patents \((n, m, n', m')\) and suppose that both firms have entered the final product market. We denote by \( p_A(n') \) the price paid by the PAE for the \( n' \) patents acquired from firm A, and we define \( p_B(m') \) analogously\(^{15}\).

Then, using the same notation as in section 4, we obtain the following payoffs from licensing and litigation:

\[
\pi_A = \pi + T(n-n', m-m') - T(m', 0) + p_A(n'), \quad \pi_B = \pi - T(n-n', m-m') - T(n', 0) + p_B(m')
\]

\[
\pi_{PAE} = T(n', 0) + T(m', 0) - p_A(n') - p_B(m').
\]

\(^{15}\)To be rigorous we should write \( p_A(n, m, m', n', s) \), but we omit the rest of the variables for the sake of exposition.
Licensing and litigation when only one firm enters

Suppose that only firms A enters the final product market. In this case, firm B technically acts as a PAE when monetizing its patents. The payoffs from licensing and litigation in this case are:

\[ \pi_A = \pi_n - T(m,0) + p_A(n'), \quad \pi_B = T(m,0) + p_B(m'), \quad \pi_{PAE} = T(m',0) - p_A(n') - p_B(m'). \]

In the next section we solve the problem of finding the equilibrium values of \( n' \) and \( m' \), which arise as the solution of a bilateral bargaining game between the operating firms and the PAE.

5.1 Patent Trade - Bilateral Bargaining

We adopt the simultaneous and symmetric Nash bargaining approach of [Horn and Wolinsky (1988)]. The PAE simultaneously bargains with firms A and B over the outcomes. An outcome corresponds to the set of patents acquired by the PAE from the producing firms and the prices at which they were bought. Let \( S_{PAE}(n', m') \), \( S_A(n', m') \), and \( S_B(n', m') \) be the payoffs from licensing in the shadow of litigation for the PAE, firm A, and firm B, respectively, after the PAE acquires \( n' \) patents from A and \( m' \) from B, at prices \( p_A(n') \) and \( p_B(m') \), respectively.

The result of each negotiation is the solution to Nash Bargaining, given the deal reached between the PAE and the other producing firm. In this section we do not assume symmetric bargain power when firms negotiate patent sales with the PAE. The bargaining power of the producing firms is \( s \) and is \( (1-s) \) for the PAE, with \( s \in [0, 1] \).

For a given profile of initial patent portfolios \( (n, m) \), with \( n + m = N \), we find the equilibrium of this bargaining game between the producing firms and the PAE. We focus on the case \( n \geq m \) since the other case is symmetric.

More formally, given that the producing firms have bargaining power \( s \), the equilibrium in the bargaining game is one in which, taking \( m' \) as given, firm A and the PAE bargain over their outcome à la Nash

\[ (n', p_A) \in \max_{z,p} (S_{PAE}(z, m') - p) - S_{PAE}(0, m'))^{1-s} (S_A(z, m') + p - S_A(0, m'))^s. \]  \hspace{1cm} (1)

Taking \( n' \) as given, firm B and the PAE bargain over their outcome

\[ (m', p_B) \in \max_{z,p} (S_{PAE}(n', z) - p) - S_{PAE}(n', 0))^{1-s} (S_B(n', z) + p - S_B(n', 0))^{s}. \]  \hspace{1cm} (2)
We denote by \( J_{A,PAE}(z,m') \) the joint surplus of firm A and the PAE from the litigation stage, when firm A transfers \( z \) patents to the PAE and firm B has sold \( m' \) patents to the PAE. Analogously, \( J_{B,PAE}(z,m') \) for firm B and the PAE. A standard result in bargaining games is the following:

**Lemma 2.** The outcome of the bilateral negotiation between an operating firm and the PAE maximizes their joint surplus, for a fixed deal between the rival firm and the PAE.

Bilaterally, an operating firm and the PAE trade patents to maximize their joint surplus. Once the allocation of patents maximizes the joint surplus, a monetary transfer splits the surplus between the parties according to their bargain power. Thus, to find the amount of patents traded between producing firms and the PAE, we need to examine the allocations that maximize the joint surplus.

**Proposition 1.** It is an equilibrium for each firm to sell its whole portfolio to the PAE in the bargaining game described above. In that equilibrium, the PAE extracts no rents from the producing firms, i.e. \( \pi_{PAE} = 0 \).

*Proof.* When a producing firm sells its entire portfolio to the PAE, the other producing firm and the PAE achieve the same joint surplus at any allocation that monetizes all patents (when possible). Suppose firm A sold its entire portfolio to the PAE in their bilateral negotiation, that is, \( n' = n \).

First, consider the case \( m > \hat{c} \). Firm B, on its own, can use its entire portfolio and get \( Vm \) from firm A, because firm A has no patents to use defensively. The PAE will use the patents acquired from firm A against firm B, to obtain \( Vn \) in licenses. Hence, firm B’s outside option is \( S_B(n,0) = V(m-n) \) and the PAE’s outside option (from the bilateral negotiation with firm B) is \( S_{PAE}(n,0) = Vn \). The joint surplus between firm B and the PAE without agreement is \( J_{B,PAE}(n,0) = Vm \). Any amount \( z > 0 \) of patents allocated from firm B to the PAE, will generate aggregate surplus \( J_{B,PAE}(n,z) \leq Vm \). When \( z > 0 \), the aggregate surplus is strictly less than \( Vm \) when \( m - z < \hat{c} \) or \( z < \hat{c} \), which means, respectively, that firm B or the PAE cannot credible sue firm A. In all other cases, the aggregate surplus is \( Vm \). Therefore, \( Vm = \max_{z \in [0,m]} J_{B,PAE}(n,z) \). An outcome of the bargain process between firm B and the PAE is \( z^* = m \) and \( p(m) = Vm \).

Now consider the case \( m \leq \hat{c} \). Firm B cannot use its patents against firm A, nor the PAE after acquiring any amount \( z \in (0,m] \). Thus, in that case, firm B and the PAE are indifferent among any outcome \( z \in [0,m] \) and price \( p(z) = 0 \).
In any case, when firm A sold everything to the PAE, it is weakly dominant for firm B to sell everything to the PAE. Analogously, when firm B is selling all its patents to the PAE, an outcome of the bilateral negotiation between firm A and the PAE is to have firm A sell all its portfolio to the PAE at price $V_n$.

When Firm A sells all its portfolio to the PAE it loses the ability to countersue. This implies that the maximum surplus that firm B and the PAE can achieve jointly, equals what firm B achieves on its own. The PAE does not offer any strategic advantage to firm B, and therefore it does not increase their joint surplus, and firm B is indifferent between selling or not. Proposition 1 shows one equilibrium outcome of the bargaining game in which both producing firms are selling their entire portfolio to the PAE. The PAE will monetize the patents, but it will not extract rents from the producing firms. The equilibrium payoffs will be:

$$\pi_A = \pi + V(n - m), \quad \pi_B = \pi - V(n - m), \quad \pi_{PAE} = 0.$$  \hspace{1cm} (3)

Depending on the size of the portfolios there are cases in which the bargaining game has multiple equilibria. The following lemmas are used to characterize the multiple equilibria. We first study the case in which firm B has enough patents to be monetized by the PAE.

**Lemma 3.** Suppose $m > \hat{c}$. In any equilibrium of the bargaining game firm B sells all of its patents to the PAE.

**Proof.** Suppose by contradiction there is an equilibrium in which firm B sells $m' < m$ and retains $k = m - m'$. Given $m'$, the strategy that maximizes the joint surplus between firm A and the PAE is for firm A to retain $\ell = \max\{0, k - \hat{c}\}$ and for the PAE to acquire $n - \ell$ patents from firm A. To show this claim, we analyze two cases: $k \leq \hat{c}$ and $k > \hat{c}$.

When $k \leq \hat{c}$, firm A does not face a direct litigation threat from firm B. But if firm A were to sue firm B, those $k$ patents would be used defensively, which is the reason why firm A is better off selling everything to the PAE to avoid the counterclaims of those $k$ patents. We distinguish the cases $m' > \hat{c}$ and $m' \leq \hat{c}$. When $m' > \hat{c}$, without an agreement between firm A and the PAE, the PAE gets $V m'$ from suing firm A with the patents acquired from firm B, and firm A gets $V(n - k)$ from firm B. Thus, the bilateral joint surplus without agreement is $J_{A,PAE}(0, m') = V m' + V(n - k) - V m'$. When $m' \leq \hat{c}$, without an agreement between firm A and the PAE, firm A gets $V(n - k)$ by suing firm B and the PAE gets 0. Thus, for any value of $m'$, the bilateral joint surplus between firm A and the PAE without agreement is
\[ J_{A,PAE}(0, m') = V(n - k). \] By selling everything to the PAE, firm A and the PAE generate their maximal joint surplus of \( J_{A,PAE}(n, m') = Vn \), which is strictly larger than \( J_{A,PAE}(0, m') \) for \( m' < m \).

When \( k > \hat{c} \) and firm A sells everything to the PAE, firm A leaves itself vulnerable to firm B’s litigation threat. However, firm A can “cancel out” this litigation threat by holding on to some patents. The minimum amount of patents that are sufficient to deter firm B from litigation is \( \ell = k - \hat{c} \). Again, we distinguish the cases \( m' > \hat{c} \) and \( m' \leq \hat{c} \). When \( m' > \hat{c} \), without an agreement between firm A and the PAE, the PAE gets \( Vm' \) from firm A, using the patents acquired from firm B, and firm A gets \( T(n, k) \) from firm B. The bilateral joint surplus without agreement is \( J_{A,PAE}(0, m') = T(n, k) \). When \( m' \leq \hat{c} \), without an agreement the PAE gets 0 and firm A gets \( T(n, k) \) as joint surplus without an agreement. Consider firm A keeping \( k - \hat{c} \) patents and selling \( n' = n - k + \hat{c} \) to the PAE. Notice that \( n > m \geq k \) implies that \( n' > \hat{c} \), so the PAE can credible monetize the patents acquired from firm A. By keeping \( k - \hat{c} \) patents, firm A effectively deters firm B from starting a lawsuit. Thus, the joint surplus between firm A and the PAE in this case is \( J_{A,PAE}(n', m') = Vn' \) which is strictly larger than \( J_{A,PAE}(0, m') \).

In fact, this is the largest joint surplus that firm A and the PAE can achieve, because selling more than \( n' \) would imply that firm B has a credible threat against firm A (which lowers the bilateral joint surplus), and selling less than \( n' \) would imply that the PAE extract less surplus from firm B.

We have shown that firm A and the PAE best respond to \( m' < m \) by playing the strategy \( n'(m') = n - \max\{0, k - \hat{c}\} \). But in this case, firm B and the PAE do not maximize their joint surplus at \( m' < m \), since \( J_{B,PAE}(n', m') = T(m', 0) < Vm \). By selling everything to the PAE, firm B and the PAE can guarantee a larger joint surplus. Therefore, selling \( m' < m \) cannot be an equilibrium.

Intuitively, when firm B holds on to some patents, firm A and the PAE have a strategy that prevents firm B from monetizing those patents. When firm A and the PAE are playing this strategy, firm B and the PAE are losing value on the patents held by firm B, since the PAE could monetize them.

Hence, when \( m > \hat{c} \) any equilibrium must have firm B selling everything to the PAE. We now study the case in which firm B’s portfolio cannot be monetized by the PAE.

**Lemma 4.** When \( m \leq \hat{c} \), in every equilibrium of the bargaining game firm A sells all of its
Proof. Suppose there is an equilibrium in which firm A sells \( n' < n \) and retains \( \ell = n - n' > 0 \). If \( \ell \leq \hat{c} \) firm A is not monetizing those patents (firm B cannot sue firm A). Thus, in an equilibrium we must have \( \ell > \hat{c} \) to increase the bilateral surplus between firm A and the PAE. But in this case, firm B and the PAE best respond by using all of firm B’s patent to countersue firm A. But this cannot be an equilibrium, because when firm B retains all its patents, firm A and the PAE maximize their joint surplus by allocating all the patents to the PAE and letting it monetize them.

If firm B does not have enough patents to be monetized by the PAE, firm A is “safe” selling all its patents to the PAE. This in turn avoids counterclaims that would be brought by firm B, had firm A sued directly.

Lemmas 3 and 4 characterize the unique equilibrium behavior of one of the firms in the game. The multiplicity arises from the different strategies that the other firm can play. The next proposition characterizes all the equilibrium payoffs.

**Proposition 2.** The equilibrium payoffs of the game are:

1. When \( m > \hat{c} \), firm B sells its entire portfolio. The equilibrium payoffs of any equilibrium are:
   \[
   \pi_A = \pi + V(n-m), \quad \pi_B = \pi - V(n-m), \quad \pi_{PAE} = 0.
   \]

2. When \( m \leq \hat{c} \), firm A sells its entire portfolio to the PAE and Firm B is indifferent between selling any amount \( m' \in [0,m] \). The equilibrium payoffs depend on how many patents firm B is selling, as they change the outside option in the bilateral bargaining of firm A and the PAE.
   \[
   \pi_A = \pi + T(n,m-m') + s[Vn - T(n,m-m')], \quad \pi_B = \pi - Vn, \quad \pi_{PAE} = (1-s)[Vn - T(n,m-m')].
   \]

Proof. In this proof we also find all the equilibria of the game.

1. When \( m > \hat{c} \), by lemma 3 any equilibrium features firm B selling everything to the PAE.

   Consider an equilibrium where firm A keeps \( \ell > 0 \) patents. Because this is an equilibrium, firm A and the PAE must get the highest joint surplus with this allocation of
patents. If firm A kept all its patents, the joint surplus from litigation between firm A and the PAE would equal $Vn$. Any other equilibrium allocation must have all the patents being monetized to achieve this maximal joint surplus $Vn$. This is the maximal joint surplus, because firm B has sold all its patents to the PAE. Therefore, for any equilibrium with $\ell > 0$ we must have $\ell \in [\hat{c}, n - \hat{c}]$. This implies that when firm A deals with the PAE there is no increase in joint surplus between firm A and the PAE. Thus, the PAE extract no rents from firm A.

But also, firm B and the PAE must maximize their joint surplus when firm B sells all its patents to the PAE. Firm B and the PAE could increase their joint surplus by “cancelling out” firm A’s litigation threat. The minimum number of patents that firm B must keep to prevent firm A to start a lawsuit is $k = \ell - \hat{c}$. If the PAE can monetize the patents bought from firm B, that is $m - k > \hat{c}$, and $k < m$, then firm B and the PAE would have a profitable deviation. By cancelling out firm A’s threat, firm B and the PAE can increase their joint surplus by $V\hat{c}$. But this cannot be an equilibrium, since by lemma (3) firm B sells all its patents in equilibrium. Hence, either firm B does not have enough patents to cancel firm A’s lawsuit ($k > m$) or, by keeping some patents, the remaining patents cannot be monetized by the PAE or firm B ($m - k < \hat{c}$). Suppose firm B has enough to “cancel out” firm A’s litigation threat, but when doing it the remaining patents cannot be monetized by the PAE, which is equivalent to $m < \ell < m + \hat{c}$. In this case, by keeping all its portfolio, firm B and the PAE have a joint bilateral surplus of $-V(n - \ell)$ which is a profitable deviation. Thus, in equilibrium it must be the case that firm B does not have enough patents to “cancel out” firm A’s litigation threat. That is, $\ell > m + \hat{c}$. When firm A keeps this large amount of patents, firm B cannot avoid the litigation threat by holding on to some patents, and therefore firm B’s outside option does not depend on the amount of patents kept by firm A.

Therefore, for an equilibrium with $\ell > 0$ to exist, it must be the case that $m + \hat{c} < \ell < n - \hat{c}$. In this equilibrium, firm B and the PAE’s joint surplus is $-V(n - m)$, and the PAE does not increase their joint surplus either, so extracts no rents from firm B.

This implies that the PAE extract no surplus in this case.

2. When $m \leq \hat{c}$ and firm B keeps $k = m - m'$ the outside option of firm A is to use its portfolio to litigate when possible, which only happens when $n - k > \hat{c}$. In this case, firm

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16This is because by holding on to $k$ patents firm B has to pay $V(n - \ell)$ to the PAE, as the PAE monetizes the patents bought from firm A, and also the PAE gets $V(m - k)$ from monetizing the patents bought from firm B. When firm B sells everything to the PAE, the joint surplus is $-V(n - m)$ as firm B gets $-Vn$ from the litigation against firm A and the PAE, and the PAE monetizes firm B patents getting $Vm$. 

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A obtains $T(n, k)$. By selling all its patents to the PAE, firm A avoids the counterclaims brought by firm B using the portfolio it withheld. Thus, the PAE monetizes all firm A’s patents and firm A does not faces a threat of litigation from firm B. Thus, the joint surplus between firm A and the PAE is in this case $V_n$. The extra surplus from selling everything to the PAE is given by $V_n - T(n, k)$, which is split according to firm A’s bargain power $s$.

Our results show the difference in the continuation payoff between the game with PAEs and without PAEs (section 4). In the figure below, we summarize our main findings, for the case $n > m$, which is the effect of the PAE on the equilibrium continuation payoffs. From the figure it is easy to see that the PAE affects the firms’ continuation payoffs only in three regions. The case $n < m$ is symmetric by changing the roles of firms A and B.

![Figure 4: Changes produced by the PAE in the firms’ continuation payoffs for a given patent portfolio $(n, m)$, with $n > m$. The payoffs are symmetric for the case $n < m$. The changes in payoffs occur only in three regions.](image)

1. **Region (I):** When $|n - m| \leq \hat{c}$, $m \leq \hat{c}$, and the PAE did not exist, firms were not suing each other. Even when $n > \hat{c}$, firm A did not have the ability to monetize its patents, because of the fear of retaliation from firm B. If the PAE exists, it allows firm A to monetize its patents (when $n > \hat{c}$) by “cancelling out” firm B’s portfolio. This strategic advantage offered by the PAE comes from its ability to avoid countersuing. Notice that although firm B cannot monetize its patents, its decision of how many patents to keep has an impact on the equilibrium payoffs, as they determine the outside option of firm A in the bilateral bargain with the PAE.
The effect of the PAE on equilibrium payoffs is to increase firm A’s payoff from 0 to $sVn + (1 - s)T(n, m - m')$, and decrease firm B’s payoffs by $Vn$.

2. **Region (II):** When $|n - m| > \hat{c}$, $m \leq \hat{c}$, and although firm A had a credible litigation without the PAE, the PAE can offer to “cancel out” firm B’s portfolio, which increases firm A’s surplus and the PAE is able to extract rents. The effect of the PAE on equilibrium payoffs is to increase firm A’s payoff from $V(n - m)$ to $V(n - m) + sVm + (1 - s)Vm'$, and decrease firm B’s payoffs by $Vm$.

3. **Region (III):** When $|n - m| \leq \hat{c}$, $m > \hat{c}$, and without PAEs, firms did not have a credible litigation threat. However, when the PAE acquires all the patents it has two individually rational lawsuits $n > \hat{c}$ and $m > \hat{c}$, against firm A and B, respectively. In this case, patent monetization generates a positive total surplus of $V(n - m)$. However, although aggregate surplus increases, firm A gets all of it and firm B loses all of it. Therefore, comparing to the case of no PAEs, the PAE increases firm A continuation payoff by $V(n - m)$, which equals the decrease in the continuation payoff for firm B. The PAE is not able to extract surplus in this case, because in equilibrium the “weak” player (the firm with fewer patents) sells everything to the PAE. This implies that firm A’s outside option is to monetize all its patents without a threat of countersuing, which is the that the PAE can offer.

4. **Other cases:** When $|n - m| > \hat{c}$, $n > \hat{c}$, and $m > \hat{c}$ firms A and B were already monetizing all its patents, and therefore the PAE does not offer any strategic advantages. And finally when $n \leq \hat{c}$, and $m \leq \hat{c}$ no firm has enough patents to start a lawsuit. Hence, the PAE does not affect the continuation payoffs.

5.2 **Entry decisions**

Analogously to the case without PAEs, as long as duopoly profits are large enough, it is a dominant strategy for both firms to enter the market. We assume in our model that the PAE approaches the firms after they enter the market. If we switch the timing, allowing the PAE to approach firms before they enter the market, the results are the same as long as $\pi$ is large enough. Results could be modified only if trading with the PAE before entering the market affect the entry decision of firms.
5.3 Continuation payoffs with PAEs

In this section we summarize the subgame equilibrium: Firms do not trade patents, they both enter, and the PAE acquire patents from the operating firms. After the patent acquisition, the firms and the PAE bargain over licenses. Proposition 2 presents all the equilibria and continuation payoffs of the game for any allocation of patents. We call $U_{PAE}^A(n, N - n)$ the continuation payoff for firm A when it discovered and patented $n$ components.

The figure below illustrates the continuation payoffs $U_{PAE}^A(n, N - n)$, derived in proposition 2, for the case $N > 3\hat{c}$. When $n \geq N - \hat{c}$ we have multiplicity of equilibrium payoffs, because firm B’s indifference affect firm A’s outside option in the bilateral bargain with the PAE. In the figure, we have selected the equilibrium in which the firm with the smallest portfolio keeps all its patents when it is indifferent. In this equilibrium selection, the PAE extracts the largest possible surplus from the operating firms.

![Figure 5: Consider a producing firm which discovers $n$ components, while its rival discovers $N - n$. Suppose that if a firm discovers less than $\hat{c}$ components, it will retain all its patents. The firm’s payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE.](image_url)

When firm A has considerable less patents than firm B, $n \leq \hat{c}$, the rival firm sells its entire portfolio to the PAE and the payoff for firm A is $\pi - V(N - n)$. Notice that the payoff in this region is unaffected by the amount of patents held by firm A, nor by the bargain...
power of the PAE. If \( n \in \left[ \hat{c}, \frac{N-\hat{c}}{2} \right] \) the PAE does not affect the continuation payoffs. When firms have similar portfolio sizes, \( n \in \left[ \frac{N-\hat{c}}{2}, \frac{N+\hat{c}}{2} \right] \) the PAEs breaks the “IP truce” and the firm with the largest portfolio is able to monetize it. The payoff for firm A in this region is \( \pi + V(2n - N) \). Again, this continuation payoff is unaffected by equilibrium selection, nor by the PAE bargaining power. Similar to a previous case, if \( n \in \left[ \frac{N-\hat{c}}{2}, N - \hat{c} \right] \) the PAE does not affect the continuation payoffs. Finally, when \( n \in [N - \hat{c}, \hat{c}] \) the equilibrium payoffs depends on the bargain power of the PAE and the equilibrium selection. Figure 5 is drawn for the most favorable equilibrium selection for the PAE, which occurs when firm B keeps all its portfolio when \( m \leq \hat{c} \). In this case, the continuation payoff for firm A is \( \pi + V(2n - N) + s(N - n) \). Notice that the bargaining power changes the continuation payoff. In particular, if \( s = 0 \) firm A’s payoff is \( \pi + V(2n - N) \), which coincides with the payoff without PAEs. When \( s = 1 \) the continuation payoff for firm A’s payoff is \( \pi + Vn \), which exactly compensates firm A for the loss of firm B. Thus, when \( N > 3\hat{c} \), the PAE is able to extract rents only when the firm have highly asymmetric portfolio sizes.

The cases \( N \in [2\hat{c}, 3\hat{c}] \) is qualitatively similar to \( N > 3\hat{c} \). The other important case is, \( N \in [\hat{c}, 2\hat{c}] \), because PAEs are ineffective to monetize patents for portfolios is similar size, because in this case firms individually have less than \( \hat{c} \) patents. The details for these two cases are in Appendix 3. For the remaining sections, we focus our analysis on the case \( N > 3\hat{c} \).

In the next section, taken the continuation payoffs as given, we study the decision problem of how much to invest in R&D to determine (stochastically) the number of patents discovered by each firm and the moment at which firms start producing the final product.

### 6 Endogenous R&D investments

In this section we study the optimal R&D investments when firms rationally anticipate the subsequent payoffs from entry, trade with the PAE, licensing and litigation. After each firm makes its investments, discoveries arrive stochastically and are patented, and once all \( N \) components are discovered, firms play their optimal strategies in the entry, licensing, and litigation stages of the game. We focus on the cases where Lemma 1 holds, and also on the continuation payoffs with PAEs depicted in figure 5, i.e. \( \hat{\pi} > N > 3\hat{c} \).

Before investing in R&D firms anticipate the continuation payoff, which depends on the number of components discovered by each firm. When PAEs do not exists, we denote by \( U(k) \) the continuation payoff of a firm that discovers (and patents) \( k \) out of \( N \) components.
(see figure 3), while its rival discovers the remaining $N-k$ components. When PAEs are allow
to interact with operating firms, analogously, we denote the continuation payoff by $U_{PAE}(k)$
(see figure 5).

Firms simultaneously make R&D investments to patent the $N$ components, anticipating the
continuation payoffs. We denote by $x$ and $y$ the R&D investments by firm $A$ and firm $B$, respectively. The cost of $z$ units of R&D is the same for both firms, given by $c_I(z)$. Firm $A$
discovers any one particular component independently with probability $p(x, y) = \frac{h(x)}{h(x)+h(y)}$, which implies a binomial distribution for the total number of discovered components.

$$P(k; x, y) = \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k}.$$ 

R&D investments not only determine the distribution of patents, but also the time at which
production begins. Since only the first version of a component is patentable, the time at
which component $i$ is available is given by $\tau_i(x, y) = \min\{\tau(x), \tau(y)\}$, where $\tau(x)$ and $\tau(y)$
are random arrival times for firm $A$ and $B$, respectively, whose distributions depend on the
R&D investments. Production can take place only when every component has been discovered,
since firms either invent and get a patent, or imitate. The time at which firms will enter the
market and produce is given by $\tau(x, y) = \max_{i=1,\ldots,N} \{\tau_i(x, y)\}$ distributed according to $F(\tau; x, y)$.
Firms discount profits at rate $r$.

### 6.1 The investment problem without PAEs

Each firm chooses its own R&D investment to maximize its expected payoff, given the in-
vestment level chosen by its rival. Since firms are symmetric, we focus on firm $A$’s problem,
which is:

$$\max_{x \geq 0} \mathbb{E}_r[e^{-r\tau|x, y}| \cdot \mathbb{E}_k[U(k)|x, y] - c_I(x).$$

Let $G(x, y)$ be the expected discounted rate, and $\Pi(x, y)$ the expected continuation payoff,

$$G(x, y) = \int_0^\infty e^{-r\tau} F(\tau; x, y), \quad \Pi(x, y) = \sum_{k=0}^{N} \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k} U(k).$$

In Appendix B, we derive an explicit formula for $G(x, y)$ and show its properties. Note that
$\Pi(x, x) = \pi$ because, given symmetric R&D investments, firms expect symmetrically each
portfolio allocation, expected licensing transfers for each firm net to zero, and the expected
reward of entering the market is $\pi$. Thus, the R&D decision problem can be written as:
\[ \max_{x \geq 0} \ G(x, y) \Pi(x, y) - c_{f}(x). \]

This formulation is a generalized rent seeking contest, because firms derive utility for a bundle of objects and their investments also determine the “size of the pie” through the discount factor. A standard rent seeking contest usually is a competition for a single prize (see for example Corchón (2007)). Competition over multiple prizes is studied in, for example Clark and Riis (1998), although their setting differs from ours, because in our formulation firms care (non-linearly) about the bundle of components they obtain.

From figures (3) and (5) we can see that the continuation payoff is weakly increasing, non-linear, and not concave. Besides this difficulty, the expected discount factor for \( N > 1 \) is not a concave function of the R&D investment (See Appendix B). As a consequence, the R&D decision problem is not generally well-behaved (in particular, not pseudo-concave) which does not allow us to use standard results for existence and comparative statics for symmetric games. However, under a stability condition similar to the one in Lee and Wilde (1980), we can show the problem has a unique interior solution \( x^{*} > 0 \) that solves:

\[ G_{x}(x^{*}, x^{*})\pi + G(x^{*}, x^{*})\Pi_{x}(x^{*}, x^{*}) = c'(x^{*}). \]  

The first order condition characterizes how the payoffs from the continuation game change the incentives to innovate. Investing one more unit of R&D has two effects. First, it brings the expected continuation payoff earlier. This effect is given by the term \( G_{x}(x^{*}, x^{*})\pi \). Because more R&D today brings this payoff sooner, it will be discounted at a smaller rate, captured by the marginal change in the discounting rate, \( G_{x} \). Second, as the firm with the largest portfolio can capture weakly more rents than \( \pi \) through licenses in the continuation stage, firms race to be the firm that discovers more components. This is represented by the second term \( G(x^{*}, x^{*})\Pi_{x}(x^{*}, x^{*}) \). The marginal gain \( \Pi_{x}(x, y) \) is positive for all \( x \) and \( y \), by first order stochastic dominance.\(^{18}\) In Appendix A we derive an explicit formula for the marginal rent seeking incentive:

\[ \Pi_{x}(x, x) = \frac{h'(x)}{h(x)} \Psi(\hat{c}, V), \]

where \( \Psi(\hat{c}, V) \) is decreasing in \( \hat{c} \) and increasing in \( V \).

\(^{17}\)As far as we know, the literature in “contests for bundles” has not been well developed yet.

\(^{18}\)\( \Pi_{x}(x, y) = \frac{\partial \Pi(p)}{\partial p} = \frac{\partial \Pi(p)}{\partial p} \frac{h'(x)}{h(x)} p(1-p). \) By properties of the binomial distribution (FOSD), and using the fact that \( U(k) \) is weakly increasing in \( k \), we have \( \frac{\partial \Pi(p)}{\partial p} > 0. \)
Our analysis will restrict attention to parameters values such that a symmetric pure strategy equilibrium exists. In some cases, a symmetric pure strategy equilibrium with positive level of investment might fail to exists. This happens when \( \pi \) is too small or \( r \) is too large, in which case there is not much of an incentive to invest in R&D since the expected discounted continuation profits are too small. For an extended discussion of existence and uniqueness of equilibrium, see Appendix D.

6.2 The investment problem with PAEs

When firms are allowed to trade patents with the PAE, the continuation payoffs are modified, as described in Proposition 2. We focus our analysis on the case \( N > 3\hat{c} \), for which the continuation payoffs are shown in figure 5. As discussed before, the PAE has two effects on equilibrium payoffs. First, the PAE is able to extract rents from the producing firms when one firm has a portfolio size smaller than \( \hat{c} \). In this case, the PAE lowers the expected discounted continuation payoff, as firms now expect to earn less than \( \pi \) in a symmetric equilibrium, and this difference is given by the amount of rents extracted by the PAE. Second, in any equilibrium of the bargaining game the effect of the PAE is to weakly increase the payoff of the firm with more components and to weakly decrease its rival, compared to the case of no PAEs (see Figure 5). The “winner” (the firm that discovered more components) gains more and the “loser” loses more, relative to the case without PAEs. More importantly, when firms’ portfolios are similar, the firm with the largest portfolio is able to monetize it, in contrast with the case of no PAE.

Let’s denote by \( U^{PAE}(k) \) the expected continuation payoff of a firm that discovers \( k \) components in presence of the PAE. We define \( S(k) \) as the difference in the payoffs from the continuation game with and without PAEs:

\[
S(k) = U^{PAE}(k) - U(k).
\]

In figure 5, \( S(k) \) corresponds to the difference between the bolded line and the gray line. Specifically, for the case \( N > 3\hat{c} \) we have:

\[
S(k) = \begin{cases} 
-Vk & k \in [0, \hat{c}] \\
0 & k \in [\hat{c}, \frac{N-\hat{c}}{2}] \text{ or } \left[\frac{N+\hat{c}}{2}, N-\hat{c}\right] \\
V(2k-N) & k \in \left[\frac{N-\hat{c}}{2}, \frac{N+\hat{c}}{2}\right] \\
V(N-k) - (1-s)V\ell, & k \in [N - \hat{c}, N]
\end{cases}
\]
where \((1 - s)\) is the bargaining power of the PAE and \(\ell\) is the amount of patents retained by the operating firm with the smallest portfolio. Notice that the multiplicity of equilibria and the bargain power only affects the the payoffs when \(n > N - \hat{c}\). The most favorable case for the PAE is when \(\ell = m\) and \(s = 0\), where the PAE is able to extract \(V(N - k)\). The least favorable case for the PAE is when \(s = 1\), where the PAE does not extract any surplus. The figure below illustrates \(S(k)\) for the case \(\ell = N - k\) and \(s \in (0, 1)\).

\[
S(k) = 0 \quad \text{for} \quad k \geq \frac{N}{2}, \quad S(k) = 0 \quad \text{for} \quad k \leq \frac{N}{2}.
\]

**Figure 6:** Difference in continuation payoffs induced by the PAE, when \(N > 3\hat{c}\).

From the figure we can see the two effects mentioned before. First, in terms of levels, the PAE increases the payoff of the “winner” and decreases that of the “loser,” which is formally given by \(S(k) \geq 0\) for \(k \geq \frac{N}{2}\) and \(S(k) \leq 0\) for \(k \leq \frac{N}{2}\). If the PAE is able to extract positive rents, it does by not completely rewarding the firm with the largest portfolio for the rents extracted from the firm with the smallest portfolio. That is, for \(k \in [N - \hat{c}, N]\), we have that \(S(k) < |S(N-k)|\), and this difference depends on the equilibrium selection and the bargaining power \(s\). Thus, when firms are ex-ante identical the expected value of \(S()\) is negative, meaning that firms expect \(\pi - E[S(k)]\) as the average continuation payoff.

Second, in terms of marginal incentives (slopes), we can see that in the region where the firms used to play the *truce equilibrium* without PAEs, which is \(\frac{N-\hat{c}}{2} \leq k \leq \frac{N+\hat{c}}{2}\), the PAE changes the payoffs strictly since \(S'(k) > 0\). However, at the extremes, the effect of the PAE is negative since \(S'(k) < 0\). We will show that in a symmetric equilibrium, since firms expect to end up closer to the middle region, the effect on incentives is always positive.

To analyze the effect of the PAE, we define \(PAE(x, y)\) to be the expected discounted difference in a firm’s equilibrium payoffs with and without PAEs, given R&D investments \(x\) and \(y\).

\[
PAE(x, y) = G(x, y) \sum_{k=0}^{N} P(k; x, y)S(k).
\]
Let $V^\text{Eq}(x, y)$ be the objective of firm A when it chooses its R&D investment in the absence of PAEs, for a given investment level $y$ of firm B,

$$V^\text{Eq}(x, y) = G(x, y)\Pi(x, y) - c_I(x).$$

In the presence of PAEs, given the R&D level $y$ of firm B, firm A solves:

$$\max_{x \geq 0} V^\text{Eq}(x, y) + PAE(x, y).$$

The effect of PAEs on incentives is given by the marginal expression:

$$PAE_x(x, y) = G_x(x, y) \sum_{k=0}^{N} P(k; x, y)S(k) + G(x, y) \sum_{k=0}^{N} P_x(k; x, y)S(k).$$

The first term, the *rent extraction effect*, reflects the marginal effect of R&D investments on the time at which the PAE’s expected distortion on the level of rents arrives. This expected distortion can be positive or negative, depending on the investment levels $x$ and $y$, as they change the mean of the distribution $P(k; x, y)$, which is $N\rho$. The second term, the *winner premium effect* is the extra reward that the firm with the largest portfolio gets by the increased patent monetization induced by the PAE.

**Proposition 3.** When $N > 3\hat{c}$, for symmetric R&D investments $x = y = x_{PAE}^*$ we have:

- **The rent extraction effect (RE) is weakly negative and equal to**

  $$RE(x_{PAE}^*) \equiv G(x_{PAE}^*, x_{PAE}^*) \frac{h'(x_{PAE}^*)}{h^2(x_{PAE}^*)} \frac{r\ln(N)}{2N+2} \sum_{k=0}^{N} \binom{N}{k} \eta(k; s) \leq 0,$$

  where $\eta(k; s) = -(1-s)V\ell_k$, $\ell_k$ is the amount of patents retained by the firm with the smallest portfolio, and $(1-s)$ is the PAE’s bargaining power.

- **The winner premium (WP) effect is strictly positive and equal to**

  $$WP(x_{PAE}^*) \equiv G(x_{PAE}^*, x_{PAE}^*) \frac{h'(x_{PAE}^*)}{h^2(x_{PAE}^*)} \sum_{k=0}^{N} \binom{N}{k} (2k-N)S(k) > 0.$$  

- **A sufficient condition for the winner premium effect to be larger than the rent extraction effect is**

  $$h(x_{PAE}^*) > \frac{r\ln(N)}{2}.$$
The previous proposition characterizes the PAE effect in a symmetric equilibrium. First, it shows that the rent effect extraction is always negative, unless $s = 1$ or $\ell = 0$. Equivalently, the PAE is unable to extract rents if it does not have any bargaining power or the firm with the smallest portfolio sells everything to the PAE. In any other case, the PAE is able to extract a positive amount of rents from the market. This implies that the PAE has a negative impact on the marginal effect of bringing the future sooner. Second, the proposition shows that the PAE has a positive impact on the rent seeking incentive. Although the continuation payoff becomes “flatter” on the extremes, it becomes “steeper” in the middle (see figure 5). In a symmetric equilibrium, being in the middle region is more likely and this outweighs the potential negative effect on incentives on the extremes. Finally, the last result in the proposition provides a sufficient condition to have an unambiguous result on the total effect of the PAE. It shows that $PAE_x(x, x) > 0$ whenever $h(x) > \frac{r \ln N}{2}$. In other words, when firms invest the same amount in R&D, the winner premium dominates the rent extraction effect when the symmetric R&D investment is larger than some threshold.

Now that we understand the marginal effect of the PAE, we can study how the equilibrium R&D investments compare in the cases with and without the PAE. Under conditions of existence and uniqueness discussed in Appendix [3] there exists a unique interior symmetric equilibrium without PAEs, $x^*$ such that

$$foc(x^*) \equiv G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) - c'_f(x^*) = 0.$$  

If we incorporate the PAE, the condition changes to

$$foc(x^*_{PAE}) + PAE_x(x^*_{PAE}) = 0$$

We also showed in the appendix that around the symmetric equilibrium the game behave as strategic substitutes. For that to be true, we needed that $h(x) > \frac{r \ln N}{2}$, but in that case we also find that the PAE effect is always positive.

**Proposition 4.** If a symmetric equilibrium with and without PAEs exist, and the equilibrium values are such that $h(x) > \frac{r \ln N}{2}$, then the equilibrium with PAEs is larger than the equilibrium without PAEs.

*Proof.* Let $x^*$ be the equilibrium without PAEs, so $foc(x^*) = 0$. By proposition [3] we know that $PAE_x(x) > 0$ for all $x \geq \tilde{x}_{PAE} = h^{-1}\left(\frac{r \ln N}{2}\right)$. Therefore, any $x_{PAE}$ such that $foc(x^*_{PAE}) + PAE_x(x^*_{PAE}) = 0$, we have that $foc(x^*_{PAE}) < 0 = foc(x^*)$. But in the region where $h(x) > \frac{r \ln N}{2}$ we showed that $foc(x)$ is strictly decreasing and therefore we must have $x^* < x^*_{PAE}$. \qed
Proposition 4 establishes that PAEs can *increase* the level of R&D investment, although they extract rents from the market.

![Best Response functions](image1)
![First Order Conditions](image2)

**Figure 7:** Effect of the PAE on the firm equilibrium. Solid lines correspond to the case without a PAE, while dotted lines correspond to the case with a PAE. The winner premium effect dominates the rent extraction effect at higher investment levels, which provides firms extra incentives to invest.

In the next section we explore when it is desirable to increase the level of R&D investment from a social welfare perspective.

### 7 Welfare Analysis

In this section we compare the symmetric equilibrium solution to what a planner would do if it could control the level of investment in each firm. Notice that with or without PAEs, the continuation game between the firms (and the PAE) is a zero-sum game with total industry profits $2\pi$. The planner does not care about the allocation of patents, as long as both firms enter, even if one of the firms owns all the patents. Hence, the first best solution, in which the planner controls the investment levels of the firms and grants a license to every firm, coincides with the second best solution, in which the planner just controls the investment levels and
once the patents are allocated among firms, they will bargain over licenses in the shadow of litigation, possibly through PAEs.

The social planner chooses investment levels \(x \) and \(y \) to maximize the total surplus generated by the commercialization of the final product. Let \(W \) be the consumer surplus generated by discovering all the components and selling the final product. Then, the planner solves

\[
\max_{x \geq 0, \ y \geq 0} (2 \pi + W)G(x, y) - c_I(x) - c_I(y).
\]

An interior solution therefore implies the conditions

\[
(2 \pi + W)G_x(x, y) = c_I'(x) \quad \text{and} \quad (2 \pi + W)G_y(x, y) = c_I'(y).
\]

By concavity of \(h\) and convexity of \(c_I\) any interior solution is symmetric. The symmetric solution for the planner’s problem is equivalent to the competitive firm best response when its rival invests zero, after relabeling the parameters values: \(\pi \to 2 \pi + W\) and \(r \to \frac{r}{2}\). Therefore, under our assumptions in Appendix D, the planner solution \(x_P > 0\) exists and its unique.

The symmetric equilibrium conditions can be written as:

\[
G_x(x_P, x_P)\pi - c_I'(x_P) + G_x(x_P, x_P)(\pi + W) = 0 \quad \text{(Planner problem)}
\]

\[
G_x(x^*, x^*)\pi - c_I'(x^*) + G(x^*, x^*)\Pi(x^*, x^*) = 0 \quad \text{(Equilibrium)}
\]

By comparing \(G_x(x, x)(\pi + W)\) and \(G(x, x)\Pi_x(x, x)\) we find when the planner invests more or less relative to the firm equilibrium.

The term \(G_x(x, x)(\pi + W)\), which we call the planner incentive, represents the marginal benefit of higher investment that is internalized by the planner but not the firms. It corresponds to the marginal expected discounted consumer surplus and payoff of the rival firm. The term \(G(x, x)\Pi_x(x, x)\), which we called competition incentive, represents the winner premium which is taken into account by firms, but not by the planner. These two effect disalign the incentives to invest between the planner and the firms. In the next lemma, we show how these two different components are ordered for different levels of R&D investments.

**Lemma 5.** The planner incentive is larger than the competition incentive if and only if \(x > x_M\), where \(x_M\) is defined as:

\[
h(x_M) = \frac{2^{N - 1}(\pi + W)r\ln(N)}{V \cdot \sum_{k:|2k-N| \geq \hat{c}} \binom{N}{k}(2k - N)^2}
\]
Proof. We show in Appendices A and D that

\[ G_x(x, x) = G(x, x) \frac{r \ln(N)}{4h(x)^2} h'(x) \quad \text{and} \quad \Pi_x(x, x) = \frac{h'(x)}{h(x)} \frac{V}{2^{N+1}} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2. \]

Substituting these in and rearranging, we obtain the condition

\[ (\pi + W)G_x(x, x) > G(x, x)\Pi_x(x, x) \iff h(x) < \frac{2^{N-1}(\pi + W)r \ln(N)}{V \cdot \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2} = h(x_M). \]

Lemma 5 shows that the planner incentive is larger than the competition incentive for levels of R&D investment smaller \( x_M \). Moreover, the comparative statics of this cutoff point are straightforward: it increases with consumer welfare increases, duopoly profits, discount rate, \( \hat{c} \), and decreases with \( V \). The intuition for this result is simple. The planner incentive only depends on bringing the payoff earlier.

Using lemma 5 we can compare the symmetric equilibrium with the planner solution.

1. **Under-investment:** If \( x^* < x_M \), then \( x^* < x_P \).

2. **Over-investment:** If \( x^* > x_M \), then \( x^* > x_P \).

Firms will underinvest only when \( x_M \) is large. The main reason why \( x_M \) is large is because \( W \) is likely to be large to capture R&D spillovers and consumer welfare. Although measuring private versus social returns of R&D is not an easy task, Jones and Williams (1998) and Hall (1996) point out that private R&D investment is lower than the social optimal level of investment. Hence, it is likely \( x_M \) is large and firms are under-investing relative to the planner. But in that case, the effect of the PAE could attenuate the under-investment problem.

8 Conclusions

This paper provided a theoretical framework to understand the effect of Patent Assertion Entities (PAEs) on the incentives for litigation and innovation. In particular, we focused on the practice of “Patent Privateering”, which is the outsourcing of patent monetization by producing firms. In our model, firms decide their level of R&D investment by looking forward to the continuation payoffs. We computed the continuation payoffs when PAEs do not exist, and we compared them to the situation where a PAE is allow to biliterally bargain with the producing firms after they entered the market.
Our analysis showed that the equilibrium effect of PAE activity, conditional on producing firms entering the product market, depended on the comparison of two effects: the rent extraction and the winner premium. The rent extraction effect reflects the disincentive to invest in R&D to bring the continuation payoff sooner, because the PAE is able to extract rents during the bargaining stage and lowers total industry profits. The winner premium effect is a rent-seeking incentive to invest, because the PAE is able to charge higher licensing fees when monetizing patents. The “winner”, the firm with the largest portfolio, is the firm that benefits from the PAE intervention, while the the firm with the smallest portfolio ends up paying higher licenses. Thus, in presence of the PAE, firms invest more resources in R&D to avoid being the “loser” firm.

Although much of the attention is given to the rent extraction effect when PAEs are evaluated, we identified another channel, the winner premium effect that goes in the opposite direction, and could overwhelm the rent extraction effect. We provided reasonable conditions under which the effect of the PAE is in fact to increase the incentives to invest in R&D. In other words, we have shown that PAEs can increase R&D incentives even when: it does not invest in R&D, it does not use the patents to produce a product, it does not have any advantage in litigation with respect to the producing firms, it lowers total industry profits, and it extracts a positive amount of rents.

The ex-ante strengthening in R&D competition induced by the PAE comes from the fact that firms anticipate better continuation payoffs if they come up ahead. The PAE is in a better bargaining position, relative to a producing company, because it cannot be countersued. An argument for why operating companies would want to avoid litigation in the first place is the fear of retaliation. This “mutually assured destruction” scenario is what leads to an ex-post inefficient equilibrium, since the firm with the largest portfolio is unable to monetize it. This happens, for example, whenever two competing firms have unequal but similarly-sized patent portfolios. If neither firm is willing to enforce its patents in court, since the expected legal costs outweigh the net value of patents (after accounting for the value of the rival’s patents in a counter-suit), then firms remain in a tacit “IP truce”. However, the PAE is able to monetize patents in these cases, because it does not fear retaliation. When the PAE disrupts this type of “IP truce”, it actually creates incentives on the margin for firms to invest more in R&D, in order to capture precisely that value which would otherwise not have been realized.

Our main result resembles to what would happen in a world with no legal transaction costs.

\(^{19}\)We have imposed that \( \hat{\pi} \geq N > 3\hat{c} \), plus an stability condition \( h(x) \geq \frac{-\ln N}{2} \) to guarantee existence of interior symmetric equilibrium.
In that case, the firm with the largest portfolio would always obtain a positive payment from its rivals. But PAEs also extract rents from the market, which lowers total industry profits. This effect is dominant when \( N \) is small relative to \( \hat{c} \).20

Finally, besides identifying these two effect when PAEs act as “patent privateers”, our main policy message is to be careful when evaluating the effect of PAEs on incentives for innovation. Policy makers should prudently evaluate their impact on incentives for innovation and also on litigation. Even when the rent extraction effect is negative, there might be other equilibrium consequences, as firms look forward, that could overweight the negative effect caused by PAEs lowering industry profits. In fact, our model shows that the best way to prevent the negative effect of PAEs on incentives is to decrease their bargain power. One way to do this, is to allow for a more competitive market of patent privateers. An empirical investigation on the impact of PAE privateers on R&D is missing in the literature. Future work should look at the incentives to invest in R&D for firms that sell patents to PAE privateers.

\[ \text{20In particular, for } N \in [\hat{c}, 2\hat{c}], \text{ the winner premium effect dissapears with similar sized portfolios.} \]
References


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Appendix A: Proofs

Proof of Lemma 2.

Proof. Consider the bilateral bargain between firm A and the PAE, taking the outcome of the negotiation between firm B and the PAE as given. Using the change of variables \( u = S_{PAE}(z, m') - p - S_{PAE}(0, m') \), the maximization problem (1) can be written as

\[
\max_{z,u} u^{1-s}(J_{A,PAE}(z, m') - J_{A,PAE}(0, m') - u)^s.
\]

The solution is \( z^* \in \arg \max J_{A,PAE}(z, m') \) and \( u^* = (1-s)[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')] \), which implies the transfer \( p = s[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')] - [S_A(z^*, m') - S_A(0, m')] \).

The agreement is incentive compatible as long as \( J_{A,PAE}(z^*, m') \geq J_{A,PAE}(0, m') \).

The following Lemma is used in several of the proofs of the propositions:

Lemma 6. Let \( f : \{0, 1, ..., N\} \rightarrow R \). Define \( f(N/2) = 0 \) if \( N \) is odd. Then,

\[
\sum_{k=0}^{N} \binom{N}{k} f(k) = \sum_{k < \frac{N}{2}} \binom{N}{k} [f(k) + f(N - k)] + \binom{N}{N/2} f(N/2).
\]

Proof. By properties of the binomial coefficient \( \binom{N}{k} = \binom{N}{N-k} \), for all \( k = 0, ..., N \). We have:

\[
\sum_{k=0}^{N} \binom{N}{k} f(k) = \sum_{k < \frac{N}{2}} \binom{N}{k} f(k) + \sum_{k > \frac{N}{2}} \binom{N}{k} f(k) + \binom{N}{N/2} f(N/2)
\]

\[
= \sum_{k < \frac{N}{2}} \binom{N}{k} f(k) + \sum_{k > \frac{N}{2}} \binom{N}{N-k} f(k) + \binom{N}{N/2} f(N/2)
\]

\[
= \sum_{k < \frac{N}{2}} \binom{N}{k} f(k) + \sum_{s < \frac{N}{2}} \binom{N}{s} f(N-s) + \binom{N}{N/2} f(N/2) \quad \text{(change of variable)}
\]

\[
= \sum_{k < \frac{N}{2}} \binom{N}{k} [f(k) + f(N-k)] + \binom{N}{N/2} f(N/2).
\]

In particular, when \( f(k) + f(N-k) = 0 \) for all \( k = 0, ..., N \), and \( f(k) < 0 \) for \( k < \frac{N}{2} \):

\[
\sum_{k=0}^{N} \binom{N}{k} f(k) \geq 0, \quad \sum_{k=0}^{N} \binom{N}{k} kf(k) > 0.
\]
Derivation of $\Pi_x(x, x)$

Lemma 7.

$$\Pi_x(x, x) = \frac{h'(x)}{h(x)} \frac{V}{2^{N+1}} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2 \equiv \frac{h'(x)}{h(x)} \Psi.$$ 

**Proof.** Let $p(x, y) = \frac{h(x)}{h(x)+h(y)}$. We have that

$$\frac{\partial}{\partial x} P(k; x, y) = \binom{N}{k} \frac{p^k}{p(1-p)}(1-p)^{N-k}(k-Np).$$

Therefore, the marginal change in the expected payoff is given by:

$$\Pi_x(x, y) = \frac{p_x}{p(1-p)} \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np) \cdot U(k).$$

Using the definition of $p$ we find $p_x = \frac{h'(x)}{h(x)} p(1-p)$. Replacing we obtain:

$$\Pi_x(x, y) = \frac{h'(x)}{h(x)} \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np) \cdot U(k).$$

Define $f_N(k) = p^k(1-p)^{N-k}[U(k) - \pi]$, and applying Lemma 6 we obtain,

$$\sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np)[\pi + f_N(k)] = \pi \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np) + ... = 0$$

$$... + \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np) f_N(k).$$

Therefore, the marginal benefit in terms of expected continuation payoff is given by:

$$\Pi_x(x, y) = \frac{V h'(x)}{h(x)} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} p^k(1-p)^{N-k}(k-Np)(2k-N).$$

In a symmetric equilibrium, $p^* = \frac{1}{2}$ and therefore the expression above equals:

$$\Pi_x(x, x) = \frac{V h'(x)}{2^{N+1} h(x)} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2 \equiv \frac{h'(x)}{h(x)} \Psi.$$

Notice that $\Psi$ measures the intensity of rent seeking incentives, and it is decreasing in $\hat{c}$, and increasing in $V$. 

$\square$
Proof of Proposition 3

Proof. Using symmetry of the payoff function we have $S(k) = -S(N - k)$ for $\hat{c} \leq k \leq N - \hat{c}$. For $k < \hat{c}$ we have $S(k) = -V k$, and for $k > N - \hat{c}$ we have $S(k) = V(N - k) - (1 - s)\ell_k$, where $\ell_k$ is the amount of patents retained by the firm with the smallest portfolio in equilibrium. Thus, for $k > N - \hat{c}$ we have that $S(k) + (1 - s)V\ell_k = -S(N - k)$.

In a symmetric equilibrium $p = \frac{1}{2}$ and by symmetry of the binomial coefficients we have that:

$$\sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}S(k) = \frac{1}{2} \sum_{k=0}^{N} \binom{N}{k} S(k),$$

and we can decompose the sum as:

$$\sum_{k=0}^{N/2} \binom{N}{k} S(k) + \sum_{k=N/2}^{N} \binom{N}{k} S(N - k).$$

Using the relation described above for different regions between $S(k)$ and $S(N - k)$ (for all $k$) we have:

$$\sum_{k=0}^{N} \binom{N}{k} S(k) = -(1-s)V \sum_{k=N - \hat{c}}^{N} \binom{N}{k} \ell_k.$$

Defining $\eta(k; s) = -V\ell_k(1-s)/N_k$ and noticing that $\eta \leq 0$, we have the result.

- We borrow some algebra from the results in “Derivation of $\Pi_x(x, x)$”. Since $S(k) \geq 0$ if $2k > N$ and non-positive otherwise, the winner incentive effect is given by

$$WP(x_{PAE}^*) = G(x_{PAE}^*, x_{PAE}^*) \frac{Vh'(x_{PAE}^*)}{2N+2h(x_{PAE}^*)} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)S(k) > 0.$$

Notice we can decompose this effect in two regions: One that increases relative to the case of no PAEs, which is the “middle region” $k \in \left[\frac{N - \hat{c}}{2}, \frac{N + \hat{c}}{2}\right]$, and one that decreases incentives $k < \hat{c}$ or $k > N - \hat{c}$. However, overall the winner premium effect is still positive.

- To show the last part of the proposition, define

$$\kappa(x_{PAE}^*) = \left[WP(x_{PAE}^*) + RE(x_{PAE}^*)\right] \cdot \left[\frac{G(x_{PAE}^*, x_{PAE}^*) \frac{Vh'(x_{PAE}^*)}{2N+1h(x_{PAE}^*)}}{1}\right].$$

We have that $\kappa(x_{PAE}^*) > 0$ if and only if

$$\sum_{k=0}^{N} \binom{N}{k} (2k - N)S(k) + \frac{r \ln(N)}{2h(x_{PAE}^*)} \sum_{k=N - \hat{c}}^{N} \binom{N}{k} \eta(k; s) > 0,$$

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which implies that $\kappa(x^*_{PAE}) > 0$ if and only if
\[
h(x^*_{PAE}) > \frac{r \ln(N)(1 - s)V \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \ell_k}{2 \sum_{k=0}^{N} \binom{N}{k} (2k - N)S(k)}.
\]

It is easy to see that
\[
0 \leq \frac{\sum_{k=N-\hat{c}}^{N} \binom{N}{k} (1 - s)V \ell_k}{\sum_{k=0}^{N} \binom{N}{k} (2k - N)S(k)} \leq \frac{\sum_{k=0}^{\hat{c}} \binom{N}{k} V k}{\sum_{k=0}^{N} \binom{N}{k} (2k - N)S(k)} \leq 1.
\]
This implies the sufficient condition in the proposition.

\[\square\]

**B Appendix B: Continuation payoffs with PAEs**

In this section we complete the analysis of the continuation payoffs for the cases $N < 3\hat{c}$. First of all, when $N < \hat{c}$ the PAE has no effect in continuation payoffs as the firms will never monetize their portfolios. Here, again we break the indifference by assuming that the firm with the smallest portfolio holds on to all its patents.

**Case $N \in [\hat{c}, 2\hat{c}]$:**

From Proposition 2, we obtained the payoffs for the different cases. If $n \in [0, N - \hat{c}]$, firm A gets $\pi - V(N - n)$. If $n \in [N - \hat{c}, \hat{c}]$, firm A gets $\pi$, since both $n$ and $m$ are less than $\hat{c}$. If $n \in [\hat{c}, \frac{N + \hat{c}}{2}]$, the PAE is able to monetize firm A’s portfolio, while firm A on its own cannot. Thus, firm A’s payoff is $\pi + sVn$. Finally, if $n \in [\frac{N + \hat{c}}{2}, N]$, firm A could monetize its portfolio, although firm B would countersue. The PAE allows firm A to avoid the countersuing, and thus firm A’s payoff is $\pi + V(2n - N) + sV(N - n)$.

[INSERT FIGURE]

Notice that this case is qualitatively different to the case presented in the paper, because the PAE is not able to break the “IP-truce” when the patent portfolios are of similar size. Therefore, the effect of the PAE is potentially negative in this case.
Case \( N \in [2\hat{c}, 3\hat{c}) \):

This case is qualitatively similar to the case presented in the body of the paper \( N > 3\hat{c} \). From Proposition 2 we obtained the payoffs for the different cases. If \( n \in [0, \hat{c}] \), firm A gets \( \pi - V(N - n) \). If \( n \in [\hat{c}, N - \hat{c}] \), \( \pi + V(2n - N) \) since the this region the PAE does not extract rents in equilibrium. If \( n \in \left[ N - \hat{c}, \frac{N + \hat{c}}{2} \right] \), the PAE is able to monetize firm A’s portfolio, while firm A on its own cannot. Thus, firm A’s payoff is \( \pi + sVn \). Finally, if \( n \in \left[ \frac{N + \hat{c}}{2}, N \right] \), firm A could monetize its portfolio, although firm B would countersue. The PAE allows firm A to avoid the countersuing, and thus firm A’s payoff is \( \pi + V(2n - N) + sV(N - n) \).

[INSERT FIGURE]

C Appendix C: Derivation of \( G(x, y) \)

In this appendix we derive an explicit formula for \( G(x, y) \) and we study its properties.

Explicit formula

Since each component has an independent exponential arrivals we have that

\[
G(x, y) = \int_0^\infty e^{-rt} N(1 - e^{-(h(x) + h(y))t})^{N-1}(h(x) + h(y))e^{-(h(x) + h(y))t} dt
\]

Given the symmetry, let’s call \( \lambda = h(x) + h(y) \), so we have

\[
g(\lambda) = N\lambda \int_0^\infty e^{-(r+\lambda)t} (1 - e^{-\lambda t})^{N-1} dt
\]

\[
= N\lambda \int_0^\infty e^{-(r+\lambda)t} \left[ \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-\lambda kt} \right] dt
\]

\[
= N\lambda \sum_{k=0}^{N-1} \binom{N-1}{k} \left( -1 \right)^k \int_0^{\infty} e^{-(r+\lambda(k+1))t} dt
\]

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\[
\sum_{k=0}^{N-1} \binom{N}{k} \frac{(-1)^k \lambda}{r + \lambda(k+1)} = 
\sum_{k=0}^{N-1} \binom{N}{k+1} \frac{(-1)^k \lambda(k+1)}{r + \lambda(k+1)} = 
- \sum_{k=1}^{N} \binom{N}{k} \frac{(-1)^k \lambda k}{r + \lambda k} \quad \text{(change of variables)}
\]

In the last step, we added and subtracted \( r \) in the numerator, and we used the fact that 
\( \sum_{k=0}^{N} \binom{N}{k} (-1)^k = 0 \).

Notice that \( g(\lambda) \) is equivalent (for \( \lambda > 0 \), and using induction) to

\[
g(\lambda) = \sum_{k=0}^{N} \binom{N}{k} (-1)^k \frac{r}{r + \lambda k} = \frac{N!}{(\frac{r}{\lambda} + 1) (\frac{r}{\lambda} + 2) \cdots (\frac{r}{\lambda} + N)} = \frac{\Gamma(N+1)}{\Gamma(N+1 + \frac{r}{\lambda})}
\]

where \( \Gamma(z) \) is the Gamma function, which is increasing and convex for \( z > 1 \). It’s easy to see that for \( N > 1 \) we have \( g(0) = 0 \), \( g'(0) = 0 \), \( \lim_{\lambda \to \infty} g(\lambda) = 1 \) and \( \lim_{\lambda \to \infty} g'(\lambda) = 0 \). Also, we can show that \( g \) is increasing and S-shaped. It is convex and then concave.

Notice that \( \ln(g(\lambda)) = \ln(N!) - \ln(\Gamma(h_N(\lambda))) \), where \( h_N(\lambda) = N + 1 + \frac{r}{\lambda} \). Thus,

\[
g'(\lambda) = -g(\lambda)\ln(\Gamma(h_N(\lambda)))' = \frac{g(\lambda)r}{\lambda^2} \frac{\Gamma'(h_N(\lambda))}{\Gamma(h_N(\lambda))} > 0.
\]

\[
g''(\lambda) = -\{g(\lambda)\ln(\Gamma(h_N(\lambda)))'\}'
= -g'(\lambda)\ln(\Gamma(h_N(\lambda)))' - g(\lambda)\ln(\Gamma(h_N(\lambda)))''
= g(\lambda) \left\{ [\ln(\Gamma(h_N(\lambda)))']^2 - [\ln(\Gamma(h_N(\lambda)))]'' \right\}
\]

After some algebra we can show that

\[
g''(\lambda) = \frac{g(\lambda)}{\Gamma^2} \left\{ [\Gamma']^2 - [\Gamma']' \Gamma - \frac{r}{\lambda^2} [\Gamma' \Gamma] \right\} \bigg|_{h_N(\lambda)}
\]

The convex region of \( g \) is where \( \left\{ [\Gamma']^2 - [\Gamma']' \Gamma - \frac{r}{\lambda^2} [\Gamma' \Gamma] \right\} \bigg|_{h_N(\lambda)} > 0 \).

**Approximation**
In order to get more tractable properties of $G(x, y)$ we will use the Stirling approximation, which is a highly precise approximation for the Gamma function, even for small values of $N$:

$$\frac{\Gamma(x + 1 + \beta)}{\Gamma(x + 1 + \alpha)} \approx \frac{\sqrt{2\pi} (x + \beta)^{x+\beta}}{\sqrt{2\pi} (x + \alpha)^{x+\alpha}} = \left(1 + \frac{\beta - \alpha}{x + \alpha}\right)^{x+\alpha+1/2} \left(1 + \frac{\beta}{x}\right)^{\beta-\alpha} \left(\frac{x}{e}\right)^{\beta-\alpha} = x^{\beta-\alpha}.$$  

Therefore, assuming that $N$ is large (larger than 6 is highly precise approximation), a good approximation is:

$$\hat{g}(\lambda) = N^{-\frac{\lambda}{2}} = e^{-r \frac{\ln(N)}{\lambda}}$$

The properties if $g$ are easily derived using this approximation:

$$\hat{g}'(\lambda) = \hat{g}(\lambda) \frac{r \ln(N)}{\lambda^2}$$

$$\hat{g}''(\lambda) = \hat{g}(\lambda) r \frac{\ln(N)}{\lambda^4} (r \ln(N) - 2\lambda)$$

It’s easy to see that $\hat{g}$ is increasing and S-shaped: Initially convex, for $\lambda < \bar{\lambda} = \frac{r \ln(N)}{2}$, and for $\lambda > \bar{\lambda}$ is always concave.

### D Appendix D: Existence and Uniqueness of Equilibrium

#### Existence

We cannot apply the standard results of existence of equilibria, despite the game being symmetric, the payoffs are continuous, and the actions are chosen from a convex and compact set\textsuperscript{21}

Consider firm A’s problem

$$\max_{x \in [0, M]} u_A(x, y) \equiv G(x, y)\Pi(x, y) - c(x)$$

\textsuperscript{21}Clearly the game is symmetric game, and the payoffs are continuous. Since $G(x, y) \leq 1$ and $\Pi(x, y) \leq \pi + NV$, no firm will choose an investment level above $M = c^{-1}(\pi + NV)$. Thus, strategies are chosen from the interval $[0, M]$.

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When $\pi > NV$ both firms always enter. We first impose a participation condition: even when firm B does not invest in R&D, firm A still wants to invest in R&D. Equivalently, if we denote A’s best response to B by $x^*(y)$, we can write this as $x^*(0) > 0$, where:

$$x^*(0) \in \text{arg} \max_{x \in [0, M]} u_A(x, 0) \equiv G(x, 0)\Pi(x, 0) - c(x).$$

As long as $\pi$ is large enough we have that $x^*(0) > 0$, since $\Pi(x, 0) = \pi + NV$ for all $x > 0$, so we have $u_A(x, 0) = G(x, 0)(\pi + NV) - c(x)$. Clearly there exists $\pi$ large enough such that the optimum is strictly positive.

Next, notice that $u_A(0, 0) = 0$ and $u_A(\infty, 0) = -\infty$. Under general conditions it can be shown that $u_A(x, 0)$ has at most two zeros in $(0, \infty)$. The next figure depicts the typical shape of $u_A(x, 0)$, showing that in fact is not quasi-concave for small values of $x$.

**Figure 8:** Simulation of the objective function for parameter values $N = 8$, $\pi = 2.5, 3.5, 4.5$, $R = 0.167$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $r = 1$, $c(x) = \frac{1}{2}x^2$, and $\hat{c} = 1.5$.

The shape of the objective function is not surprising. Intuitively, when firm B invests 0, firm A trades off the investment cost against the benefit of an earlier arrival of the continuation.

---

22 $u_A(x, 0) = 0$ is equivalent to $\pi + NV = e^{r\ln(N)}$ $c(x) \equiv K(x)$, and $K(\cdot)$ is always positive and its derivative is zero whenever $\frac{c'(x)h^2(x)}{c(x)h'(x)} = \frac{2\ln(N)}{2}$. If there a unique $x$ (or none) that satisfies this equation, then by continuity we can have at most two solutions. This is the case, for example, when $h(x) = x^\alpha$ and $c(x) = x^\beta$ with $\alpha < 1 < \beta$. 

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payoff, $\pi + NV$. When $x$ is small, firm A pays the investment cost and receives almost no benefit, since discoveries arrive far in the future. This is why the payoff decreases below zero for some small $x$. When $x$ is larger than some threshold, the time at which all the components are discovered is significantly reduced, and the continuation payoff becomes significant and not so heavily discounted, so the firm has incentives to invest. Finally, for relatively large values of $x$, increasing $x$ even more will not improve firm’s profits because the gains from discovering faster are relatively smaller than the cost of investment.

Next, consider the general problem when $y > 0$. Firm A now also faces a rent-seeking effect, as firms compete to get more components than their rivals. Moving in the opposite direction is the “free riding” benefit that firm A gets when firm B invests more, because the continuation payoff arrives earlier.

To understand how $x^*(y)$ changes with $y$, we can compute the cross partial derivative

$$\frac{\partial^2 u_A(x, y)}{\partial x \partial y} = G_{x,y}(x, y)\Pi(x, y) + G_x(x, y)\Pi_y(x, y) + G_y(x, y)\Pi_x(x, y) + G(x, y)\Pi_{x,y}(x, y)$$

Re-arranging terms, we can show that:

$$G_{x,y}(x, y) = G(x, y)r \ln(N)\frac{h'(x)h'(y)}{(h(x) + h(y))^2} \left[r \ln(N) - 2h(x) - 2h(y)\right],$$

$$G_x(x, y)\Pi_y(x, y) + G_y(x, y)\Pi_x(x, y) = \frac{G(x, y)r \ln(N)h'(x)h'(y)}{(h(x) + h(y))^2h(x)h(y)} \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}(k-Np)U(k)[h(y)-h(x)]$$

and

$$\Pi_{x,y}(x, y) = -\frac{h'(x)h'(y)}{h(x)h(y)} \sum_{k=0}^{N} \binom{N}{k} p^k(1-p)^{N-k}[(k-Np)^2 - Np(1-p)]U(k).$$

Next, we can combining these expressions to sign $\frac{\partial^2 u_A(x, y)}{\partial x \partial y}$. In particular, since we are interested in a symmetric equilibrium, we can study the local behavior when $x = y$. In a symmetric equilibrium, we have:

$$G_{x,y}(x, x) = G(x, x)r \ln(N)\frac{h'(x)^2}{(2h(x))^4} \left[r \ln(N) - 4h(x)\right] < 0 \iff h(x) > \frac{r \ln(N)}{4}$$

$$G_x(x, x)\Pi_y(x, x) + G_y(x, x)\Pi_x(x, x) = \frac{G(x, x)r \ln(N)h'(x)^2}{4h(x)^4} \Psi[h(x) - h(x)] = 0$$

$$\Pi_{x,y}(x, x) = -\frac{h'(x)^2}{h(x)^2} \sum_{k=0}^{N} \binom{N}{k} \frac{1}{2N} \left[\left(\frac{k-N}{N} - \frac{N}{4}\right)^2 - \frac{N^2}{4}\right] U(k) = 0$$

(by Lemma [\text{Lemma}])
Combining the 3 terms above, in a symmetric equilibrium we have

\[ \frac{\partial^2 u_A(x,x)}{\partial x \partial y} = G_{x,y}(x,x)\Pi(x,x) \]

and hence

\[ \text{sign} \frac{\partial^2 u_A(x,x)}{\partial x \partial y} = \text{sign} \left( r \ln(N) - 4h(x) \right) \]

Therefore, as long as the intersection of the best response and the 45 degree line occurs at \( x \) such that \( h(x) > \frac{r \ln(N)}{4} \), the local behavior in the symmetric equilibrium is such that \( \frac{\partial^2 u_A(x,x)}{\partial x \partial y} < 0 \). Since we have a symmetric game, this condition implies that locally around the region where \( x = y \) the investments \( x \) and \( y \) are strategic substitutes: when \( y \) increases, the best response \( x^*(y) \) decreases.

The figure below depicts the typical shape of the best responses when the parameter \( \pi \) is large enough so the condition \( \hat{\pi} > N > 3\hat{c} \) is satisfied.

![Best Response functions](image)

**Figure 9:** Simulation of the best responses for parameter values \( r = 1, N = 8, \pi = 5, R = 0.167, \beta = 0.8, h(x) = x^\alpha, \alpha = 0.8, c(x) = \frac{1}{2}x^2, \hat{c} = 1.5. \)

Notice there is a unique symmetric equilibrium where firms exert a positive level of R&D. Next, we show that whenever the symmetric equilibrium involves a level of investment large enough, there is a unique symmetric equilibrium.

**Condition for uniqueness**

We show that when there exists a symmetric equilibrium with positive level of investment \( x^* \) satisfying \( h(x^*) \geq \frac{r \ln(N)}{2} \), then it is unique. We know that any interior symmetric equilibrium
Generally, there are at most two positive levels of investment such that

\[ G_x(x,x)\pi + G(x,x)\Pi_x(x,x) = c'(x). \]

Define \( foc(x) = G_x(x,x)\pi + G(x,x)\Pi_x(x,x) - c'(x) \). Using the approximation for \( G(x,x) \) and the computation of \( \Pi_x(x,x) \) (derived in Appendix XXX) we have

\[ foc(x) = e^{-\frac{r \ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[ \frac{\pi r \ln(N)}{2h(x)} + \Psi \right] - c'(x). \]

Taking derivative we obtain:

\[ foc'(x) = e^{-\frac{r \ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[ \left( \frac{r \ln(N)}{2h(x)} - 1 \right) \left( \frac{r \ln(N)h'(x)}{2h^2(x)} - \pi + \Psi \right) - \frac{\pi r \ln(N)}{2h(x)} + \frac{h''(x)}{h'(x)} \right] A(x) - c''(x), \]

where \( A(x) = \frac{\pi r \ln(N)}{2h(x)} + \Psi \). Rearranging terms we obtain:

\[ foc'(x) = e^{-\frac{r \ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[ \left( \frac{r \ln(N)}{2h(x)} - 1 \right) \left( \frac{r \ln(N)h'(x)}{2h^2(x)} - \pi + \Psi \right) - \frac{\pi r \ln(N)}{2h(x)} + \frac{h''(x)}{h'(x)} \right] A(x) - c''(x), \]

Notice that for any \( x \) such that \( \frac{r \ln(N)}{2h(x)} < 1 \) we have that \( f'(x) < 0 \). Let \( \bar{x} \) be such that

\[ h(\bar{x}) > \frac{r \ln N}{2}. \]

Hence, there can be only one symmetric equilibrium with a level of investment such that \( h(\bar{x}) > \frac{r \ln N}{2} \).

Notice that \( foc(x) = 0 \) does not imply that \( x \) is an equilibrium. For example, \( foc(0) = 0 \) and we showed that \( x^* = 0 \) is not an equilibrium for \( \pi \) large enough. Depending on the parameters, there might be other candidates for equilibrium. For any \( x \) such that \( foc(x) = 0 \) and \( foc'(x) > 0 \), \( x \) cannot be an equilibrium, since locally around \( x \) there are profitable deviations, as the condition \( foc'(x) > 0 \) implies that \( x \) is a local minimum of \( G(x,y)\Pi(x,y) - c(x) \) at \( y = x \). Generally, there are at most two positive levels of investment such that \( foc(x) = 0 \), as depicted in the figure below: one where \( foc'(x) > 0 \), and one where \( foc'(x) < 0 \). In this case, the unique equilibrium with positive level of investment is where \( foc(x) = 0 \) and \( foc'(x) < 0 \).
For a large set of parameter values the conditions presented in the previous discussion are satisfied. Moreover, for those parameters we find that there is a unique equilibrium and it is symmetric. In the figure below we show comparative statics on the parameters $\pi$ and $r$. When $\pi$ increases, the symmetric equilibrium features larger levels of investment, as expected. When $r$ changes, we obtain a non-monotonic behavior in the equilibrium level of R&D. The intuition is simple: When $r$ is very small, firms are very patient and they do not heavily discount profits. Hence, the effect that dominates is the rent-seeking behavior of firms. As $r$ starts to increase, discounting provides an extra incentive for firms to invest. However, when $r$ is very large, the present discounted value of entering the market is small, even when a firm wins the race for every component.
Figure 11: Comparative statics on the parameters $\pi$ and $r$. Parameter values $N = 6$, $R = 1$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $c(x) = \frac{1}{2}x^2$, $\hat{c} = 1.5$, with $r = 1$ (on the left) and $\pi = 10$ (on the right).

**Non existence of equilibrium with positive investment**

As is standard in rent seeking games, we cannot guarantee that the investments are strategic substitutes for all parameter values. For example, when $r$ is high or when $\pi$ is low the best responses might be strategic complements everywhere, or in some region.

In the first example we have increased the value of the discount rate $r$. Firms discount so heavily the payoff after entry, that a firm finds it worth investing only when its rival has also invested sufficient resources in R&D. Moreover, in this case R&D investments are strategic complements, and when firm B increases its R&D investment, firm A will slowly increase its own R&D investment. In fact, the best response looks like the best response of a firm that simply maximizes $G(x, y)\pi - c(x)$. That is, firms are more concerned with their effect on bringing the payoff sooner than with competing for rents. In the existence section, we ruled out this case by assuming that $r$ is small and $\pi$ is large, since for those parameters we know the best response to $y = 0$ is $x^*(0) > 0$ and not $x^*(0) = 0$. When $r$ is large, so the payoff from entry is still heavily discounted, but the competition effect is significant, investments are both strategic substitutes and complements in different portions of the best response function. In these cases, there is no equilibrium with positive level of investment.

For intermediate values of the parameter $r$, we can find cases in which we have a discontinuity in the best response (since we cannot guarantee that the objective function is pseudo-concave with respect to the firm’s own decision variable). This is because the rent seeking effect becomes relevant all of a sudden, after the rival has invested a small amount in R&D. Thus, the other firm can still ‘catch up’ and compete against its rival.
Figure 12: Simulation of the best response functions for $r = 2.5$ (on the left) and $r = 5$ (on the right), with parameter values $N = 6$, $\pi = 10$, $R = 2$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $c(x) = \frac{1}{7}x^2$, and $\hat{c} > 2$. 