Coase, Antitrust, and Merger Simulation with Nonlinear Prices

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March 14 2014   PRELIMINARY DRAFT NOT FOR QUOTATION

We thank Ken Heyer, Gregory Pelnar, Sam Peltzman and Steven Salop for comments.
Abstract

This article makes three points. First, our entire notion in antitrust regarding deadweight loss needs to be reconsidered. For example, in the absence of transaction costs, there is no deadweight loss. Therefore, the presence of transaction costs must be taken into account in any analysis. Second, it then immediately follows that two key issues in any antitrust analysis are a) whether transaction costs change as a result of the antitrust action under scrutiny, and b) whether or not they change, how the existence of nonlinear pricing alters the antitrust analysis. Point a) says that a lowering in transaction costs to allow the use of a more efficient pricing mechanism is an efficiency that must be accounted for in any antitrust analysis. Point b) is a recognition that there is an incentive to engage in nonlinear pricing when there is market power and so any antitrust analysis done under the alternative and usual assumption of uniform pricing could be in error. Third, since transaction costs influence the ability of various coalitions of consumers, distributors, and manufacturers to form, cooperative game theory can provide some insight into what situations might give rise to the creation and exploitation of market power. Moreover, it can unify all of antitrust analysis whether the behavior is vertical or horizontal. The paper develops and implements a merger simulation model with nonlinear pricing to illustrate its points.
I. Introduction

Coase’s 1960 article altered fundamentally how most economists think about externalities and government intervention. Coase made the point that in a world with well-defined property rights and no transaction costs, parties would always wind up at an efficient point, thereby eliminating externalities and the need for government intervention. But Coase’s main point was that we do not live in such a world and that by the assignment of property rights, a government can influence transaction costs and thereby the ability of the economy to reach an efficient point. Therefore, a government should assign property rights in order to enable the economy to reach an efficient solution. Or, more precisely, the efficient solution is one that takes account of transaction costs and that an efficient solution will depend on the efficient assignment of property rights. Stigler only half-jokingly described Coase’s article and in particular his first result -- the one involving no transaction costs -- as 60 pages of pareto-optimality said very slowly. Stigler’s point is that since a bigger pie is always preferred to a smaller one, Coase’s point is obvious if one defines “no transaction costs” to mean that one obtains efficiency. (Stigler would be the first to admit that Coase’s point was not, at least initially, obvious to anyone. See Stigler (2003).) We explore in this article what the implications of Coase’s insights are for antitrust and show how Coase’s insights mean that we should refocus much or at least some of our economic analysis of the antitrust issues related to mergers and market power.

This article makes three points. First, our entire notion in antitrust regarding deadweight loss needs to be reconsidered. For example, in the absence of transaction costs, there is no deadweight loss. Therefore, the presence of transaction costs must be taken into account in any analysis. Second, it then immediately follows that two key issues in any antitrust analysis are a) whether transaction costs change as a result of the antitrust action under scrutiny, and b) whether or not they change, how the existence of nonlinear pricing alters the antitrust analysis. Point a) says that a lowering in transaction costs to
allow the use of a more efficient pricing mechanism is an efficiency that must be accounted for in any antitrust analysis. Point b) is a recognition that there is an incentive to engage in nonlinear pricing when there is market power and so any antitrust analysis done under the alternative and usual assumption of uniform pricing could be in error. Third, since transaction costs influence the ability of various coalitions of consumers, distributors, and manufacturers to form, cooperative game theory can provide some insight into what situations might give rise to the creation and exploitation of market power. Moreover, it can unify all of antitrust analysis whether the behavior is vertical or horizontal.

This paper is organized as follows. Section II examines the foundational model of antitrust in which a firm (or group of firms) raises price above marginal cost, restricts output, and harms consumer and total welfare. For example, the famous Williamson diagram (Williamson (1968)) is still the way mergers are thought of with the deadweight loss from raising price offset in part or in total by the efficiency gain from lowered costs. But Coase’s insight forces us to focus on transaction costs in our analysis. What prevents any firm with market power from eliminating the deadweight loss typically associated with it? How does competition affect that transaction cost and how will that transaction cost be altered by a change in market structure or by certain conduct such as the imposition of vertical restrictions? Section II explores these questions and shows that only by answering them can we hope to understand the antitrust implications of a merger or of specific conduct that comes under antitrust scrutiny. To illustrate the importance of transaction costs in an antitrust analysis, Section III focuses on the typical merger simulation used in merger analysis. That analysis assumes uniform pricing in a Bertrand model of competition and ignores the ability of firms (both pre-merger and post-merger) to eliminate deadweight loss with nonlinear pricing. We show how that analysis can be entirely misleading. To do so, we develop and implement a model of merger simulation with nonlinear pricing—a merger simulation model that we do not believe is in the literature. Using this model, we show how an analysis of a merger can
change, sometimes significantly, even with only a small departure from the standard assumption of uniform Bertrand pricing. Such departures which enhance efficiency are not only predicted by Coase’s insights but also are consistent with the empirical observations in many industries. Finally, Section IV describes a more general view of how antitrust harm can arise, closely related to cooperative game theory. The key insight is that transaction costs explain which groups are likely to form and are able to exploit other groups. Although the use of cooperative game theory in industrial organization has for the most part been overtaken by the use of non-cooperative game theory (the work of Telser (1987,2006) and Roth (2008) are notable exceptions), we will see that its use provides a unified way of understanding how antitrust should treat horizontal and vertical behavior. We also explain how that approach can be usefully applied to the current controversy over how to treat standard setting organizations and their ability to influence royalty rates on standard essential patents.

II. Coase, Transaction Costs, and Deadweight Loss

As explained in the Introduction, Coase makes the point that since everyone theoretically can be made better off when there is more output, it has to be the case that the economy is always at an efficient point as long as there are no transaction costs, by which he must mean that there are no impediments to reaching that efficient point. But that is not true in the presence of transaction costs, a point that Coase also stressed. For example, consider the standard diagram showing deadweight loss from monopoly pricing. The assumption is that under competition price equals marginal cost, while under monopoly it is set above marginal cost. Why? That pricing result is optimal for the monopolist only if it is too costly for the monopolist to figure out a way to price discriminate against its customers. That might sometimes be a good assumption, but not always. For example, in many business to business transactions, the price terms are individually negotiated, often pursuant to complex contracts. Why in
such cases should we think that an analysis based on the assumption of uniform linear pricing will produce useful insights? Indeed, what flows from thinking about transaction costs is that there will be an incentive to deploy (at a cost) pricing mechanisms to enable discriminatory pricing to individual customers. Anytime one sees the standard deadweight loss triangle, instead of the antitrust official bemoaning its existence, he should say to himself – wow, look at that opportunity for some firm to figure out how to adopt more sophisticated pricing mechanisms in order to make profits. There is an incentive for the firm to capture that deadweight loss and increase its profits, thereby eliminating the deadweight loss. Now, of course, although that well describes the incentive of a firm to make money and eliminate deadweight loss, it does not help the consumers whose welfare could fall as a result of the elimination of the deadweight loss. This observation then brings to the fore whether the appropriate role of antitrust is to maximize total or consumer surplus -- a topic much debated but not resolved. See Heyer (2006) for a discussion. We do not focus on that question in this paper but instead adopt the criteria of total surplus to illustrate our points, but our analysis can be straightforwardly altered to accommodate any objective function of the antitrust authority.¹

The implication of Coase’s insights is that an antitrust analyst should pay attention to the transaction costs of avoiding deadweight loss from market power whenever he evaluates an antitrust issue. This means that if transaction costs permit it, nonlinear pricing will be used and so an analysis that ignores that fact has the potential to mislead. This also means that the antitrust analyst should determine whether those transaction costs change in any way as a result of the antitrust action being examined.

¹ For a discussion of this topic, see also Carlton and Heyer (2008).
The relevant transaction costs for a firm that wants to engage in nonlinear pricing are those costs that the firm must bear above what it would bear if it just engaged in uniform linear pricing. A firm that wants to charge nonlinear prices must possess knowledge about buyers’ willingness to pay (or knowledge about the distribution of such willingness). The acquisition of that knowledge can be costly. Especially in today’s environment of enormous data bases, that knowledge can often be purchased, or accumulated over time. Mergers today are often motivated by a desire for firms to combine and exploit their complementary databases so as to be able to engage in more sophisticated pricing. Moreover, the complicated software algorithms that firms often use to price products based on customer characteristics are costly to build. Just think about the yield management systems that hotels and airlines use as an example. All of these costs need to be considered in any social efficiency calculation. But even if a firm knows the different willingnesses to pay of buyers, the firm must still exert costly effort to restrict arbitrage, otherwise the nonlinear pricing will be undone. The required effort may well depend on market structure. Let me illustrate this last point with several examples, each of which is interesting in its own right since each is related to many situations that arise routinely in antitrust. Each of the examples shows that the ability to practice nonlinear pricing can be influenced by antitrust actions.

a) Merger to Monopoly

Consider a model of a dominant firm with a competitive fringe. Suppose that the competitive fringe has constant returns to scale at $10. The dominant firm charges $10 even though its constant marginal cost is below $10. Suppose the consumer surplus is $20 and the deadweight loss from the $10 price is $20. Consider now what happens if it costs $25 to implement a discriminatory pricing mechanism in the absence of the competitive fringe, but that it costs $50 in the presence of the competitive fringe because it is more costly to prevent arbitrage when the competitive fringe exists. This could occur
because with lots of firms, it might be hard for the dominant firm to charge some customers a high price at the same time that it is charging other customers a low price, when the customers can always buy from the competitive fringe. In other words, the transaction cost of price discrimination is very high in the presence of the competitive fringe but lower in its absence. Absent the merger, there is not an incentive for the dominant firm to implement the pricing mechanism (the cost of doing so, $50, exceeds the maximum possible gain of $30). However, a merger that eliminates the competitive fringe could allow the monopolist to capture as profit the deadweight loss of $10 plus the consumer surplus of $20 and it would be profitable to do so since the pricing mechanism costs only $25 in the absence of the fringe. In such a situation, it is even possible for the total consumer surplus to rise post-merger if the price discrimination is not perfect. This example highlights that it is not just the change in market power, but also the change in the transaction costs of implementing various pricing mechanisms, that will matter for efficiency. Moreover, there is good reason to believe that the ability to implement various nonlinear pricing mechanisms will depend on market structure that can be affected by merger.

b) Merger to Monopoly with Efficiencies

As another example, consider a merger to monopoly of two firms that create some productive efficiency. One standard analysis is to ask by how much the price under monopoly will rise compared to the price under duopoly. The analyst will see whether the efficiency gain offsets the harm from the loss of competition. But that analysis holds constant the pricing mechanism, assuming uniform pricing both pre- and post-merger. But why is that a good assumption? Following Coase, we know that the outcome with uniform pricing can be improved upon from society’s viewpoint as long as the transaction costs are not too high for such a pricing mechanism, and we know that all parties can be made better off under such a mechanism. For example, as the previous example showed, it can sometimes be possible to use more sophisticated pricing mechanisms post-merger because of the elimination of competition.
Suppose it is possible to use two-part tariffs rather than uniform pricing post-merger because of lowered transaction costs. Might that not be relevant to determining the desirability of the merger? We have never seen such a merger simulation involving nonlinear pricing, but in the next section we implement such a simulation and show that it can give very different answers regarding the desirability of a merger compared to our standard merger simulation, especially in the presence of production efficiencies.\(^2\) One reason is that production efficiencies are most important when output is large. So if a merger will lead to efficiencies and to a more sophisticated pricing mechanism, the combined effect can lead to a large expansion of output so that the use of the more sophisticated pricing can significantly alter the evaluation of the merger compared to an analysis that assumes uniform pricing.

So far, we have given two examples involving horizontal mergers. The same principles apply to vertical mergers. We give two examples. The first involves a common concern related to vertical foreclosure, while the second involves a combination of vertical and horizontal concerns.

c) Vertical Merger to Foreclose

Suppose that an input monopolist sells to several downstream firms that compete amongst themselves. The monopolist buys one of the downstream firms. The other downstream firms complain that the merged firm will have an incentive to foreclose them from (or raise their costs for) the input in order to benefit the downstream division of the vertically integrated firm. There are numerous models of such foreclosure in the literature. See e.g. Ordover, Saloner and Salop (1990). Most, if not all, of the models involving vertical foreclosure postulate some inefficient pricing between firms and then show, ____________________________

\(^2\) Section III compares a merger simulation for three cases: pre-merger pricing is uniform, post-merger pricing is uniform; pre-merger pricing is uniform, post-merger pricing is a two-part tariff; and pre-merger pricing is a two-part tariff, post-merger pricing is a two-part tariff.
depending on the details of the assumptions (e.g., Cournot v. Bertrand), that the vertical integration can cause harm through foreclosure. But the analyst should ask what merger-generated changes make foreclosure more likely than it was pre-merger. If pre-merger the input monopolist could have contracted with one “independent” downstream firm (e.g., the firm that it is merging with), told the firm “hey, I will advantage you relative to your rivals, so let’s share the profits”, then there would be NO gain from the vertical integration. Suppose that the market conditions pre-merger were such that there were lots of very detailed and complicated contracts involving terms defining non-linear pricing and exclusion of rivals, then why should one assume that exclusion would occur post-merger when it does not pre-merger? The only reason would be that the transaction cost of the exclusionary contract that I have just described becomes easier to carry out when the firm is integrated. But why? If that cannot be answered then the regulatory or antitrust authority should be skeptical of the foreclosure argument.3 Indeed, one might think that, at least sometimes, it is easier to get away with an exclusionary contract (which is hard to observe) compared to avoiding detection of the possibility of the exclusionary act when there is the full scrutiny of a regulatory or antitrust agency that is evaluating a vertical merger and can impose conditions to prevent exclusion, especially if there is a regulatory body such as the FCC involved that is less constrained by economic principles in its actions than, say, an antitrust authority.

3 One example of this might be the vertical integration of a content provider into satellite distribution. Other competitors to the satellite provider such as cable companies might complain about possible foreclosure of desirable program content. But then the relevant question is why is this foreclosure easier post-merger than pre-merger, especially in an industry where exclusionary contracts are not unusual. See, for example, the report of Carlton et al. related to Newscorp’s acquisition of Direct TV (FCC 2003) where Carlton appeared as an expert for Newscorp.
d) Horizontal merger leading to vertical antitrust concerns

As a final example showing how transaction costs of implementing a price mechanism can change as a result of some antitrust act, consider a horizontal merger that will have vertical implications. Suppose that there is a monopolist selling to many buyers. Because of either a lack of information or the difficulty in preventing arbitrage, the monopolist can charge only uniform monopoly pricing to the many buyers. Suppose now that several of the buyers merge to form one large firm. In that case, it is easy to imagine a situation in which the large buyer can negotiate with the monopolist while the many small buyers cannot because it is too costly. The large buyer may be able to credibly commit not to engage in arbitrage, succeed in obtaining a lower marginal price than it had previously paid, and split gains with the monopolist of its output expansion through some lump-sum payment (i.e., nonlinear pricing is used for the largest buyer). The analysis of the consequences of the creation of a large buyer through merger must take into account the changed pricing mechanism that results as a consequence of the merger. The outcome could raise total surplus overall, lead to an expansion of output, and could in principle lead to a harm to the small buyers. If for example, the small buyers have a lower demand elasticity than the large buyer, the small buyers could see their prices rise after the merger. It is problematic to require that a merger, in order to pass antitrust scrutiny, must make each buyer better off (or no worse off), and we would settle for a total surplus approach, though others may differ.\footnote{We note that almost every merger likely makes some consumers worse off. Think of an airline merger where post-merger the firm moves a plane from a low demand route to a high demand route. The consumers on the low demand route may be harmed. Yet, we suspect that the Department of Justice would not block such a merger solely for that reason.}
There is an interesting antitrust wrinkle in this last example. Suppose that the buyers are not final consumers but instead purchase an input from the monopolist and then compete with each other in an output market. Then the higher marginal input price to the small buyers obviously could enhance the market power that the large buyer has in the output market. If the presence of small firms in that output market prevents the large buyer from pricing efficiently (price discriminating), then the elimination of the small buyers through high input prices could lead to even higher profits for our large buyer than would otherwise occur from just the elimination of competition because the large buyer also becomes better able to price discriminate. If so, then the large buyer and the input monopolist can coordinate and destroy the small buyers, leading to a monopoly in the output market. Then those monopoly profits in the output market can be split between the now two monopolists -- the input- and output-monopolists.5

Notice that even if all input buyers – small and large – could coordinate and bargain efficiently with the input monopolist, the large buyer would refuse because he can make more money by striking a bargain with the input monopolist to drive out the small buyers. Suppose that we consider the gain to the large buyer from having the monopolist charge a high price to its rivals, the small buyers. Part of that gain comes from the fact that the advantaged large buyer will be able to exercise monopoly power over

5 This example does not violate the “one monopoly profit” theorem. Instead, it illustrates that when there is the ability of the input monopolist to create an output monopoly that can price discriminate, this will lead to higher profits than if the output market is characterized by uniform pricing. One can view the example as the input monopolist creating the transaction technology to allow the output monopolist to price discriminate.
its consumers in a way not possible before.\textsuperscript{6} By eliminating the competitive constraints on the pricing mechanism in the large buyer’s market, the input monopolist can cause the large buyer’s profits to increase with the result that the input monopolist can capture some of that extra profit for itself. Of course, we would need to investigate why there is market power in that output market, whether the market power creates deadweight loss, what coalitions can form and how costly it is to form, but it is easy to see that the best way for our input monopolist to exploit his market power could be to skew competition in downstream markets, favor the large buyer, enable the large buyer to price discriminate (through the weakening of competitive constraints on pricing by the small buyers), and share in some of the increased downstream profits. Whether this is the best way to exploit his power will again depend on all the relative transaction costs of dealing with the various coalitions. The key point is that a central issue in many antitrust matters is how the conduct under antitrust scrutiny alters the ability to exploit the existing and/or newly-created market power. We rarely, if ever, think of things that way. We should, but don’t, pay attention to how the various transaction costs of organizing groups both horizontally and vertically may change as a result of a changed market structure.\textsuperscript{7}

III. Merger Simulation with Nonlinear Pricing

\textsuperscript{6} Of course, those downstream consumers would, following Coase, want to eliminate the deadweight loss, and could pay to undo this arrangement. To keep it simple, we will assume that such arrangements engender such high transaction costs as to eliminate their feasibility.

\textsuperscript{7} Another classic example of how market structure can alter the price mechanism used is the case of vertical integration forward by an input monopolist into an end product with a low elasticity of demand. The vertical integration allows price discrimination to occur by preventing arbitrage. See Carlton and Perloff (2005).
The previous section described how the form of the pricing mechanism, and how it is altered by some antitrust event, could matter to an antitrust analysis. The usual procedure in merger analysis is to assume uniform pricing both before and after the antitrust event. In this section, we illustrate how that approach can lead to erroneous conclusions. We begin by using a merger simulation model as it is typically used in practice. Specifically, we begin with a demand system and then solve for the resulting Bertrand equilibrium pre- and post-merger and compare the two results. We then show what happens if we assume that either pre- or post-merger the firm is able to engage in nonlinear pricing. To keep it simple, we assume a simple form — a fixed fee plus variable linear pricing. We fully understand that one could adopt more complicated schemes — indeed our logic of the previous section suggests one should explain, not assume, why such schemes are not being used. But since our main point is to illustrate the implications of altering the assumptions of uniform pricing, we think our approach suffices to make our point. Moreover, although a two-part pricing structure is a simplified form of nonlinear pricing, we are unaware of any implementation of a merger simulation with two-part tariffs, so we need to figure out how to implement that form of nonlinear pricing in a merger simulation model. This section does that. We do, however, urge future research on the use of more complicated pricing schemes and regard our simulations as illustrative, but not necessarily as establishing general theorems.
The economics literature has recognized the frequency of nonlinear pricing, including those of two-part tariffs, and has studied their properties. For example, if consumers are homogenous in their demands, it is optimal for firms to set the usage fee equal to the marginal cost of incremental units and then earn profits through the fixed fee. When consumers are heterogeneous in their demands, the analysis becomes more complicated. In the usual case where the marginal consumer consumes less than the average consumer, it is optimal for a firm to set the usage fee above the marginal cost. However, if the marginal consumer buys more than the average consumer, it may be optimal to set the usage fee below marginal cost. If the firm incurs a cost to acquire a customer, then the access fee, in the usual case, may or may not exceed that cost, depending on the elasticity of demand and consumer heterogeneity. We use the pricing mechanism of a two-part tariff in our simulations to illustrate our key points of what can happen when nonlinear pricing is allowed.

We develop a model with two vertically differentiated firms in which consumers first choose which firm’s differentiated product to consume (or the outside good) and then choose how much to consume. We use this model to address several questions:

8 For example, mobile phone contracts often consist of a monthly fee, a monthly data limit, and then an overage fee that increases with the incremental usage above the monthly data limit. See, e.g., Armstrong and Vickers (2001), Carlton and Perloff (2005), Holmes (1989), and Rochet and Stole (2002). We note that two-part tariffs are not necessarily the optimal pricing mechanisms when the buyers are firms that compete with each other and have U-shaped average cost curves. See Ordover and Panzar (1982).

9 See Carlton and Perloff (2005), Appendix 10A.
• How do the welfare implications of a merger change when firms pre- and post-merger are able to set two-part tariffs versus when they are each constrained to set a single price?

• To what extent can a change in pricing technology (e.g., a merged firm can set a two-part tariff whereas the stand-alone firms can set a single price) be considered a potential benefit of a merger, and when is that benefit most likely to alter the analyst’s evaluation of the desirability of the merger?

a) Model

We assume that two firms are vertically differentiated and each offers a product that consumers can choose to consume in multiple units. Once a consumer chooses a firm, he must purchase only from that firm. Each firm sets a fixed fee $T$ and a variable fee $p$. For example, a consumer may pay his mobile phone carrier a fixed monthly fee plus a fee based on monthly usage. Consumers are assumed to be heterogeneous in the utility they derive from consuming each product.

The model has the following elements:

• $p$ is the per-unit usage fee,

• $T$ is the fixed fee,

• $N$ is the number of consumers,

• $Q(p, T)$ is the total amount of the product demanded,

• $\alpha \in [\alpha, \bar{\alpha}]$ is a parameter that indexes consumers and $f(\alpha)$ is the number of consumers of type $\alpha$,

• $S(p, \alpha)$ is the consumer surplus of customer type $\alpha$ in the absence of a fixed fee,

• $q(p, \alpha)$ is the demand curve of customer type $\alpha$,
• $m$ is the marginal cost of an incremental unit of output, and

• $M$ is the marginal cost of an incremental consumer.

We first consider the pricing incentives of two independent firms. We then consider the pricing incentives of a single firm setting the prices for both products.

i. Two Independent Firms

Suppose that there are two firms indexed by $(1, 2)$. Assume these firms are vertically differentiated such that consumers with type $\alpha \in (\alpha_1, \overline{\alpha})$ choose Firm 1 and consumers with type $\alpha \in (\alpha_2, \alpha_1)$ choose Firm 2, where $\alpha_1$ and $\alpha_2$ are determined endogenously. Consumers with $\alpha < \alpha_2$ choose to consume the outside good.

The profit function for Firm $j$ is given by:

$$\pi_j = (T_j - M_j)N_j(p_j, T_j) + (p_j - m_j)Q_j(p_j, T_j).$$  \hfill (1)

The cut-off points, $\{\alpha_1, \alpha_2\}$, are then characterized by:

$$S_1(p_1, \alpha_1) - T_1 = S_2(p_2, \alpha_1) - T_2,$$

and

$$S_2(p_2, \alpha_2) = T_2.$$  \hfill (2)

Equation (2) indicates that a consumer of type $\alpha_1$ is indifferent between choosing product 1 and product 2. Equation (3) indicates that a consumer of type $\alpha_2$ is indifferent between choosing product 2 and the outside good.

The numbers of consumers choosing Firms 1 and 2, respectively, are then given by:

$$N_1(p_1, T_1) = \int_{\alpha_1}^{\overline{\alpha}} f(\alpha) d\alpha,$$

and

$$N_2(p_2, T_2) = \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha.$$  \hfill (4)
\[ N_2(p_2, T_2) = \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha. \quad (5) \]

The quantities demanded are given by:

\[ Q_1(p_1, T_1) = \int_{\alpha_1}^{\alpha_2} q(p_1, \alpha) f(\alpha) d\alpha, \quad \text{and} \]

\[ Q_2(p_2, T_2) = \int_{\alpha_1}^{\alpha_2} q(p_2, \alpha) f(\alpha) d\alpha. \quad (7) \]

The firms’ first-order conditions are:

\[ (T_j - M_j) N_{j,p} + Q_j + (p_j - m_j) Q_{j,p} = 0, \quad \text{and} \]

\[ (T_j - M_j) N_{j,T} + N_j + (p_j - m_j) Q_{j,T} = 0, \quad (8) \]

where \( X_{j,p} \) is the partial derivative of the quantity associated with product \( j \) with respect to \( p \) and \( X_{j,T} \) is the analogous value with respect to \( T \). Equation (8) represents four equations in four unknowns \((T_1, T_2, p_1, p_2)\) and determines the equilibrium to the Bertrand pricing game. In the Appendix, we simplify these first order conditions.

ii. One Firm Producing both Products

Consider a merger between Firm 1 and Firm 2, where the combined firm continues to produce both products. The profit function is:

\[ \pi = (T_1 - M_1) N_1(p_1, T_1) + (p_1 - m_1) Q_1(p_1, T_1) \]

\[ + (T_2 - M_2) N_2(p_2, T_2) + (p_2 - m_2) Q_2(p_2, T_2). \quad (9) \]

The first order conditions are:
We use Equations (8) and (10) to evaluate the change in prices (and the resulting changes in welfare) that arises from a merger of Firms 1 and 2.

b) Simulations

To illustrate the impact of the pricing technologies on the merger simulation results, we assume specific functional forms for demand and costs and the distribution of consumers. Specifically, we assume that consumer types are distributed $\alpha \sim U(0,1)$. We assume that demand is linear and is characterized by $q(p_j, \alpha) = X_j \alpha - p_j$ such that:

- $q(p_1, \alpha) = 12\alpha - p_1$, and
- $q(p_2, \alpha) = 5\alpha - p_2$.

Marginal costs are assumed to be constant with $m_1 = 5, m_2 = 1, M_1 = 5$, and $M_2 = 1$.

We consider three scenarios, differentiated by the pricing technology available to the firms:

- In the first scenario involving uniform pricing, we restrict firms to set $p = 0$. In other words, we only allow firms to charge a fixed access fee.\(^{10}\)

- In the second scenario also involving uniform pricing, we restrict firms to set $T' = 0$. In other words, we only allow firms to charge a usage fee.\(^{11}\)

\(^{10}\) Such pricing commonly occurs with, for example, cable television (subscribers pay a fixed monthly fee and can watch unlimited content) and health clubs (members pay a monthly membership fee and can use the gym as much as they like).
• In the third scenario, we allow firms to set both $p$ and $T$.

We first assume that the hypothetical merger results in no productive efficiencies. In this case, the merger will automatically reduce welfare as long as there is some competition that gets eliminated between the firms, holding the pricing technology constant. It is nonetheless instructive to examine the merger effects under different pricing technologies. Table 1 below reports the merger simulation results pre- and post-merger for the two types of uniform pricing and for the two-part pricing.

The results illustrate a number of points. First, for any given market structure, moving from uniform tariffs (of either form) to two-part tariffs expands output and increases consumer and total surplus. This result occurs because, as expected, the additional pricing flexibility of the two-part tariff gives the firm the ability to eliminate some of the deadweight loss that arises from the distortion in usage pricing that arises from the deviation of price from marginal cost. So, for example, Total Surplus in cols. (1) and (3) is always below its value in col. (5). The same is true for cols. (2) and (4) compared to (6). Second, for any market structure, constraining the variable price to be equal to zero (substantially below marginal cost, especially for Firm 1) introduces substantial distortions by causing consumers to over-consume. For example, in Column (1), consumers of product 1 consume more than 11 units per customer (divide $Q_1$ by $N_1$) while, in Column (3), consumers of product 1 consume approximately three units per customer and, in Column (5), consumers of product 1 consume approximately five units per customer. Consequently, total surplus when $p = 0$ is substantially lower than in the other two scenarios. Third, when $T = 0$, we observe the standard distortion from imperfect competition. Both firms set the variable price somewhat higher than the marginal costs. Nonetheless, welfare is substantially higher than in the case where $p = 0$. Fourth, under the assumption that the same pricing technology is used pre- and post-

\footnote{Such pricing is the commonly assumed uniform per-unit pricing.}
merger, a merger of Firms 1 and 2 reduces welfare regardless of the pricing technology \( \text{e.g.}, \) compare Columns (1) and (2), Columns (3) and (4), and Columns (5) and (6)). However, the amount of the reduction in surplus depends on the pricing technology. For example, the pre-merger total surplus is similar in the case of \( p = 0 \) (Column (3)) and two-part tariffs (Column (5)), $2.90 and $3.39, respectively. But total surplus declines by more in the case of \( T = 0 \), $0.38 ($2.90-$2.52), than it does in the case of two-part tariffs, $0.18 ($3.39-$3.21).

<table>
<thead>
<tr>
<th>(1) Uniform tariff ( p = 0 )</th>
<th>(2) Uniform tariff ( T = 0 )</th>
<th>(5) Two-part tariff</th>
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</thead>
<tbody>
<tr>
<td>Stand-alone</td>
<td>Merged</td>
<td>Stand-alone</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>( p_2 )</td>
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<td>$0.00</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>$66.23</td>
<td>$68.15</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>$8.23</td>
<td>$8.65</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.99</td>
<td>1.00</td>
</tr>
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<td>0.83</td>
</tr>
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</tr>
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<td>( \pi_2 )</td>
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</tr>
<tr>
<td>Total Surplus</td>
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<td>$1.00</td>
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</table>

Table 1: Illustrative Results with No Efficiencies

This last result suggests that efficiencies are likely to have a differential impact on the desirability of a merger depending on the pricing technology of the firm. To illustrate this point, we assume that the hypothetical merger reduces the cost of producing product 1 by $0.25. Table 2 below reports the merger simulation results pre- and post-merger for the two types of uniform and two-part pricing. A
merger efficiency of this magnitude is sufficient to increase total surplus in the case of two-part tariffs. But total surplus declines in the case of uniform tariffs.\textsuperscript{12}

Most importantly, to the extent that the merger changes the pricing technology for the reasons discussed in the prior section, such an effect can generate an additional merger benefit. For example, start from the case of \( T = 0 \). The pre-merger total surplus is $2.90 (Table 1, Column (3)). With no efficiencies or change in pricing technology, total surplus declines to $2.52 (Table 1, Column (4)). The $0.25 efficiency raises the post-merger welfare to $2.86 (Table 2, Column (4)), but the merger still reduces total surplus. However, further incorporating a switch to two-part tariffs increases total surplus to $3.55 (Table 2, Column (6)), making the merger look beneficial. This example illustrates that the welfare impact of lowering the transaction costs of using a more efficient pricing mechanism is potentially quite important, especially when efficiencies are involved. Indeed, it can change the entire conclusion about the benefits of the merger.

\textsuperscript{12} The direction of the effect depends on the parameters of the problem. In some cases, a merger reduces welfare by more in the case of two-part tariffs and in other cases, such as the one illustrated here, a merger reduces welfare more in the case of uniform tariffs.
The illustrative merger simulations here demonstrate the basic point that the predictions of standard merger simulation models (holding everything else constant) can vary substantially depending upon the pricing technology available to the firms. In some cases, a merger that is harmful under one pricing regime is beneficial under another regime.

### IV. Transaction Costs and Coalitions

Virtually all models and analyses that we have seen that analyze anticompetitive conduct make the implicit assumption that certain transaction costs are low and certain ones are high. Most industrial organization economists make these assumptions naturally when performing an antitrust analysis and
likely don’t think about them much. For example, imagine a situation where lots of firms are competing. Although it is well understood that those firms might well gain if they could coordinate and behave as a cartel, the transaction costs of doing so might be so high as to preclude that outcome, even in the absence of antitrust laws prohibiting such behavior. However, if there were to be a merger among lots of these firms, then presumably a common pricing authority within a firm could enable prices of the formerly independent firms to be “coordinated” easily. That is the underlying intuition for why merger policy worries about increased concentration leading to higher prices is implicitly based on an underlying assumption regarding transactions costs. That reasoning, of course, makes lots of sense but we would like to recast what may appear obvious in order to highlight how the recasting can help identify quite clearly certain circumstances when antitrust harms are most likely to occur. To be concrete, we will illustrate our point by examining three areas where it is understood how certain actions can sometimes generate antitrust harm: vertical restrictions on distributors with exclusive dealing, collective action by buyers achieved through the formation of a firm, and standard-setting.

Let us begin by introducing some slightly cumbersome notation. Let m be a set consisting of all the manufacturers in the industry, let d be the set of all the distributors, and let c be the set of all consumers of the product. Let M be the set consisting of all subsets of m, D be the set of all subsets of d, and C be the set of all subsets of c. We want the reader to now consider all possible coalitions of manufacturers, distributors, and consumers -- that is, a grand set of coalitions consisting of some manufacturers, some distributors, and some consumers. Each such coalition has associated with it a transaction cost of organizing. The theory of the core teaches us that under certain circumstances in a competitive equilibrium there is no coalition that can form that can improve the well-being of its members.\(^\text{13}\) In the

\(^{13}\) See, e.g., Telser (1987, 2006).
typical exposition of the theory of the core, transaction costs are taken as zero, so that using Coase’s reasoning in the case of zero transaction costs, the competitive equilibrium must be an efficient point. We now want to introduce positive transaction costs to explain how an antitrust harm can occur.\textsuperscript{14} We have already seen one standard instance in the example where transaction costs of coordinating price among many horizontal competitors are high so that a merger could lower those transaction costs of coordinating, thereby leading to elevated pricing. It makes sense in that setting to assume that each of the many consumers might not find it worth their while to form coalitions to negotiate with a firm with market power and return the equilibrium to an efficient point. Let us see when else transaction costs can help identify an anticompetitive possibility.

a) Vertical Restrictions -- the Case of Exclusive Dealing

In any model where vertical restrictions are used to create an anticompetitive harm, it has to be the case that the participating distributors and manufacturing firm are collectively better off while consumers (and perhaps others) are worse off as a result of the restriction. Consider the standard example of foreclosure by a dominant firm of its rivals by the use of exclusive dealing. As the story goes, there are only a few distributors, there are scale economies, and the dominant firm locks up the major distributors through exclusive dealing contracts. This can be thought of as the distributors forming a coalition with the dominant firm to exclude rivals to the dominant firm, thereby allowing the dominant firm to remain a dominant firm and reap profits, some of which he can share with his coalition partners,

\textsuperscript{14} The notion that it is costly to form coalitions and that cost should be a focus of study figures prominently in the literature on public choice and regulation (Olson (1965), Stigler (1971)). It has been less important in understanding how a firm forms as an agent for its buyers or how anticompetitive coalitions form.
Exactly how much the dominant firm has to share with the distributors will depend on how important the marginal distributor is and that could range from zero to a lot. But a moment’s thought will reveal that the harmed buyers could form a coalition and bribe the distributors to enable entry of competing manufacturers. That of course sounds pretty hard to do—high transaction costs of organizing lots of small consumers into a group. But not so fast. Why can’t a distributor or potential entrant manufacturer act on behalf of its future customers and engage in the transaction and create competition? I think the best answer is that that is complicated to do because consumers are not willing (or it is too costly) to bind future consumers to the negotiating firm. It is the difficulty of dealing with future customers that makes transaction costs especially high. That is why exclusionary conduct can sometimes succeed. Indeed, I suspect that transaction costs are an important reason for why it is easier to exclude entrants than it is to exclude established firms with established customer relations. Indeed, if dealing with future consumers creates high transaction costs, then one should expect that entry deterrence will work best when the current product is durable and sold (not rented), when there are scale economies, and when consumption of any individual consumer is uncertain. In those cases:

15 I have adhered to the standard story line of how exclusive dealing can create antitrust harm. A slight modification could be to include the cooperation of a large buyer whose presence could support new entry. Instead of sponsoring entry, the large buyer is able to obtain a low enough price so that he is effectively sharing in the monopoly profits created for the incumbent, by the large buyer’s unwillingness to purchase from a new entrant. Aghion and Bolton (1987) use a mechanism much like this one.

16 See Rasmusen et al. (1991) and Segal and Whinston (2000).

17 Selling a durable good is like locking up a consumer’s consumption today and into the future.
circumstances, the transaction cost of guaranteeing a profitable entry may become high and therefore exclusionary conduct can successfully deter entry.

b) The Firm as a Negotiating Agent for a Collective of Buyers – Credit Cards

The previous example just made the point that a firm can act as a collective bargaining agent for its customers. That means that a firm can be a way for what would otherwise be an illegal collective to form and negotiate prices on behalf of its “customers” and perhaps harm either suppliers or other consumer groups. Let us give an example which comes from the literature on two-sided markets.

Suppose all the rich buyers in the United States band together to form the “rich buying club”. The club goes to all the major stores and says that their members will boycott any store that it fails to recognize the club. Furthermore, whenever any member of the club buys something, the retail store is required to make a small payment -- say 1% of the purchase price -- to the club. The club then keeps some of the money for itself and gives some to its club members. Suppose further that the club says that a requirement of any store that recognizes the club is that the store must charge club members and non-club members the exact same retail price. Without that restriction, the 1% club fee would likely be passed along at least in part to club members at the point of sale in terms of an elevated retail price. With the price restriction in place, the store must recover any increased costs of dealing with club members from all of its customers -- a form of average cost pricing. Another way to view what has happened as a result of the operation of the club with its restrictions is that the members of the “rich buying club” have forced the stores to place a “tax” on club and non-club buyers and send that tax revenue to the club that then distributes it to its members, after keeping some of it for itself. Our description in this example is pretty close to how a credit card company works where the rewards to card holders is analogous in my example to the return of some of the 1% fee to the club members, with
the buyers who use debit or cash being analogous to the non-club members. We do not want to get into a debate as to whether this behavior on the part of the credit card companies can ever be justified based on benefits the card company creates for the firms accepting its card.\textsuperscript{18} Our only point is that calling something a “firm” should not immunize its actions from antitrust scrutiny if its effect is equivalent to forming a coalition of agents with market power who have the ability to place a monopoly tax on others.\textsuperscript{19} Firms can be viewed as a low transaction cost way of organizing buyers.

c) Standard Setting Organizations

One of the clearest examples of viewing antitrust through the lens of formation of coalitions is to focus on standard setting organizations (SSOs). An SSO is an institution whose express purpose is to allow the formation of coalitions to agree upon and promulgate standards. A typical SSO can consist of industry members that include both creators of intellectual property and users of intellectual property. The SSO, for example, can define the characteristics that every smartphone must meet in order to be “standard compliant”. Failure to be “standard compliant” can mean zero sales. The owner of a patent whose functionality is incorporated in the standard is said to have a “standard essential patent” (SEP).

\textsuperscript{18} There is an enormous literature now on this topic pro and con. There have been massive amounts of litigation in the US and in many foreign countries regarding credit cards. Carlton has worked for Discover and also for retail stores and plaintiffs adverse to credit card companies. Keating has worked for major credit card companies. Because of numerous actions by regulatory and antitrust authorities, including settlement of private litigation, the features of our example may no longer apply to credit cards in certain parts of the world.

\textsuperscript{19} Similar concerns can arise for, \textit{e.g.}, insurance companies who can act as negotiating agents for many policy holders.
The incorporation of patented functionality into a standard may confer additional market power on the owner of the SEP when prior to the setting of the standard there were competing functionalities, any one of which may have sufficed to create a useful alternative standard. Once the standard is set, each manufacturer of a standard compliant product must pay a royalty to the patent owner or be subject to a patent infringement suit. The owner of a SEP does not typically commit \textit{ex ante} to any particular royalty rate for his patent when used pursuant to the standard. The obvious antitrust concern is that the owner of the SEP can demand high royalties because his patent has been incorporated into a standard.

To address this antitrust concern, SSOs typically require their members to agree to charge FRAND (fair, reasonable, non-discriminatory) rates. There has been significant litigation about what these terms mean, but the main point we wish to bring out here is that this is a problem of coalition formation. To be voted into a standard, the SSO must reach some consensus among current members as to how to proceed. Let’s suppose that there are two substitutable patents either of which could, \textit{ex ante}, be included in the standard to produce equally valuable standards. Suppose the existing manufacturers go to inventor 1 and say “we will vote you as the standard, provided you charge us a low royalty. Moreover, for later entrants who will compete against us, you must charge them a high royalty rate. In other words, we will grant you market power that you can exploit against our future (currently non-voting) rivals and consumers, and by charging us a low rate you will be sharing some of those profits with us.” The key point to notice is that if future rivals are not members of the SSO or not important members, this is a way in which it is easy to form a coalition to raise rivals’ costs. Even if there is no explicit agreement about what the \textit{ex ante} royalty rate will be, as long as it is understood that the royalty rate will be higher for later entrants, this arrangement can succeed in allowing a consortium of manufacturers and a patent holder to exploit future consumers who likely also are not well-represented in the SSO. It is possible to prevent this type of antitrust problem by interpreting the “ND” (non-
discriminatory) part of FRAND in a particular way. If “ND” is interpreted to mean that all manufacturers should be treated similarly if they are “similar”, then as long as one interprets current and subsequent entrants to be “similarly situated” then this particular exercise of market power is avoided. The beauty of the example of the standard setting organization is that it starkly shows that coalitions are needed to both pass the standard and to exploit others. The exploiting coalition can consist of manufacturers, holders of intellectual property, and some consumers, while the victims of the exploitation can be future entrants and some consumers.

V. Conclusion

As Coase observed long ago, whenever there is deadweight loss, there is a profitable incentive for private market transactions to eliminate it. Transaction costs prevent the complete elimination of deadweight loss. The standard analysis in antitrust of deadweight loss as arising from uniform pricing is likely to be inaccurate in many settings where pricing is more complicated than uniform pricing. For example, it is well-known that in many business-to-business transactions, prices are individually negotiated and can be nonlinear. When analyzing any antitrust conduct, one should use the actual pricing mechanisms used with and without the conduct under analysis in order to properly assess the antitrust consequences of the conduct. Failure to do so can lead to erroneous results, as the simulation results with nonlinear pricing showed. The lowering of transaction costs to enable more refined pricing mechanisms can be thought of as providing a possible efficiency consequence of various antitrust conduct. The observation that it is transaction costs of forming various coalitions that impedes the elimination of deadweight loss leads to the realization that all antitrust harm -- horizontal and vertical --

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20 This is precisely the approach Carlton and Shampine (2013) adopt. See also Gilbert (2011).
arises from coalitions of some groups with low transaction costs forming to exploit the left-out industry participants who face higher transaction costs of organizing into their own coalitions.

References


Appendix A  Derivation of Equilibrium Equations to Bertrand Competition with Two-Part Tariffs

Below we derive the optimality conditions with respect to the fixed prices $T$ and the usage prices $p$. We show how to express each derivative in Equations (8) and (10) as functions of all known parameters and the four unknown choice variables. The merger simulation then simply solves the four equations derived from Equations (8) and (10).

- Derivatives of surplus functions with respect to $p$’s (holding $T$ fixed) are:

  $\alpha_{1,p_1} = -\frac{S_{1,p_1}}{(S_{1,\alpha_1} - S_{2,\alpha_1})}$,

  $\alpha_{1,p_2} = -\frac{S_{2,p_2}}{(S_{2,\alpha_1} - S_{1,\alpha_1})}$,

  $\alpha_{2,p_1} = 0$, and

  $\alpha_{2,p_2} = -\frac{S_{2,p_2}}{S_{2,\alpha_2}}$.

- Derivatives of surplus functions with respect to $T$’s (holding $p$ fixed) are:

  $\alpha_{1,T_1} = \frac{1}{(S_{1,\alpha_1} - S_{2,\alpha_1})}$,

  $\alpha_{1,T_2} = \frac{1}{(S_{2,\alpha_1} - S_{1,\alpha_1})}$,

  $\alpha_{2,T_1} = 0$, and

  $\alpha_{2,T_2} = \frac{1}{S_{2,\alpha_2}}$.

- Derivatives of $N$ functions with respect to $p$’s (holding $T$ fixed) are:
Derivatives of $N$ functions with respect to $T$’s (holding $p$ fixed) are:

\[ N_{1,p_1} = -f(a_1)\alpha_{1,p_1} = \frac{S_{1,p_1}}{(S_{1,a_1} - S_{2,a_1})} f(a_1), \]

\[ N_{1,p_2} = -f(a_1)\alpha_{1,p_2} = \frac{S_{2,p_2}}{(S_{2,a_1} - S_{1,a_1})} f(a_1), \]

\[ N_{2,p_1} = f(a_1)\alpha_{1,p_1} - f(a_2)\alpha_{2,p_1} = -\frac{S_{1,p_1}}{(S_{1,a_1} - S_{2,a_1})} f(a_1), \text{ and} \]

\[ N_{2,p_2} = f(a_1)\alpha_{1,p_2} - f(a_2)\alpha_{2,p_2} = \frac{S_{2,p_2}}{S_{2,a_2}} f(a_2) - \frac{S_{2,p_2}}{(S_{2,a_1} - S_{1,a_1})} f(a_1). \]

\begin{itemize}
  \item Derivatives of $N$ functions with respect to $T$’s (holding $p$ fixed) are:
  \end{itemize}

\[ N_{1,T_1} = -f(a_1)\alpha_{1,T_1} = -\frac{1}{(S_{1,a_1} - S_{2,a_1})} f(a_1), \]

\[ N_{1,T_2} = -f(a_1)\alpha_{1,T_2} = -\frac{1}{(S_{2,a_1} - S_{1,a_1})} f(a_1), \]

\[ N_{2,T_1} = f(a_1)\alpha_{1,T_1} - f(a_2)\alpha_{2,T_1} = \frac{1}{(S_{1,a_1} - S_{2,a_1})} f(a_1), \text{ and} \]

\[ N_{2,T_2} = f(a_1)\alpha_{1,T_2} - f(a_2)\alpha_{2,T_2} = \frac{1}{(S_{2,a_1} - S_{1,a_1})} f(a_1) - \frac{1}{S_{2,a_2}} f(a_2). \]
• Derivatives of Q functions with respect to \( p \)'s (holding \( T \) fixed) are:\(^{21}\)

\[
Q_{1,p_1} = \int \frac{\partial q(p_i, \alpha)}{\partial p_1} f(\alpha) d\alpha - \frac{q(p_1, \alpha_1)}{p_1} f(\alpha_1) \alpha_{1,p_1} = \frac{Q_1}{p_1} \epsilon_{11} + \frac{S_{1,p_1}}{(S_{1,\alpha_1} - S_{2,\alpha_1})} q(p_1, \alpha_1) f(\alpha_1),
\]

\[
Q_{1,p_2} = \int \frac{\partial q(p_i, \alpha)}{\partial p_2} f(\alpha) d\alpha - \frac{q(p_1, \alpha_1)}{p_2} f(\alpha_1) \alpha_{1,p_2} = \frac{Q_1}{p_2} \epsilon_{12} + \frac{S_{2,p_2}}{(S_{2,\alpha_1} - S_{1,\alpha_1})} q(p_1, \alpha_1) f(\alpha_1),
\]

\[
Q_{2,p_1} = \int \frac{\partial q(p_2, \alpha)}{\partial p_1} f(\alpha) d\alpha + \frac{q(p_2, \alpha_1)}{p_1} f(\alpha_1) \alpha_{1,p_1} - \frac{q(p_2, \alpha_2)}{p_1} f(\alpha_2) \alpha_{2,p_1} = \frac{Q_2}{p_1} \epsilon_{21} - \frac{S_{1,p_1}}{(S_{1,\alpha_1} - S_{2,\alpha_1})} q(p_2, \alpha_1) f(\alpha_1), \quad \text{and}
\]

\[
Q_{2,p_2} = \int \frac{\partial q(p_2, \alpha)}{\partial p_2} f(\alpha) d\alpha + \frac{q(p_2, \alpha_1)}{p_2} f(\alpha_1) \alpha_{1,p_2} - \frac{q(p_2, \alpha_2)}{p_2} f(\alpha_2) \alpha_{2,p_2} = \frac{Q_2}{p_2} \epsilon_{22} + \frac{S_{2,p_2}}{(S_{2,\alpha_1} - S_{1,\alpha_1})} q(p_2, \alpha_2) f(\alpha_2) - \frac{S_{2,p_2}}{(S_{2,\alpha_1} - S_{1,\alpha_1})} q(p_2, \alpha_1) f(\alpha_1),
\]

where

\[
\epsilon_{11} = \int \frac{p_1}{Q_1} \frac{\partial q(p_1, \alpha)}{\partial p_1} f(\alpha) d\alpha,
\]

---

\(^{21}\) Note that the cross-derivatives of demand are equal to zero. In other words, conditional on choosing product \( i \), the price of product \( j \) does not impact consumption.
\[ \epsilon_{12} = \int_{\alpha_1}^{\bar{\alpha}} q_1 p_2 \frac{\partial q(p_1, \alpha)}{\partial p_2} f(\alpha) d\alpha = 0, \]

\[ \epsilon_{21} = \int_{\alpha_2}^{\alpha_1} q_2 p_1 \frac{\partial q(p_2, \alpha)}{\partial p_1} f(\alpha) d\alpha = 0, \text{ and} \]

\[ \epsilon_{22} = \int_{\alpha_2}^{\alpha_1} q_2 p_2 \frac{\partial q(p_2, \alpha)}{\partial p_2} f(\alpha) d\alpha. \]

- Derivatives of \( Q \) functions with respect to \( T' \)'s (holding \( p \) fixed) are:

\[ Q_{1,T_1} = -q(p_1, \alpha_1) f(\alpha_1) \alpha_{1,T_1} = \frac{-1}{(S_{1,\alpha_1} - S_{2,\alpha_1})} q(p_1, \alpha_1) f(\alpha_1), \]

\[ Q_{1,p_2} = -q(p_1, \alpha_1) f(\alpha_1) \alpha_{1,p_2} = \frac{-1}{(S_{2,\alpha_1} - S_{1,\alpha_1})} q(p_1, \alpha_1) f(\alpha_1), \]

\[ Q_{2,p_1} = q(p_2, \alpha_1) f(\alpha_1) \alpha_{1,p_1} = \frac{1}{(S_{1,\alpha_1} - S_{2,\alpha_1})} q(p_2, \alpha_1) f(\alpha_1), \quad \text{and} \]

\[ Q_{2,p_2} = \frac{1}{(S_{1,\alpha_1} - S_{2,\alpha_1})} q(p_2, \alpha_1) f(\alpha_1) - \frac{1}{S_{2,\alpha_2}} q(p_2, \alpha_2) f(\alpha_2). \]