Ethical Motives for Strategic Voting

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Abstract

We propose an alternative approach to modeling strategic voting in large elections. In particular, we examine strategic voting in the presence of ethical voters, who follow a group-welfare maximizing voting rule. This rule may require them to vote strategically, that is, to vote for a candidate that is not their most preferred. The model delivers comparative statics that link the degree of strategic voting with various electoral parameters. We show that strategic voting is non-monotonic in the popularity of the Condorcet loser. This observation reconciles the apparently contradicting empirical evidence and is robust to more general specifications of the model.

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1 Introduction

In a three-candidate election where one of the candidates is a Condorcet loser, it is sometimes in the interest of the voters to vote against their true preferences to prevent the Condorcet loser from winning.\footnote{A voter supports candidate $i$ if $i$ is her most favored candidate.} This phenomenon is typically referred to as strategic voting. The intuitive rationale for such a behavior is that a voter should not “waste” his vote on candidates who have little chance of winning.

There is strong evidence of strategic voting in various elections. Many empirical studies on the topic have analyzed British elections. UK politics is conducive to strategic voting because the opposition to the main center-right party is divided between two center-left parties. The evidence from UK elections shows that the extent of strategic voting ranges from 5% to 17%, as measured by the proportion of strategic votes (see Galbraith and Rae (1989), Heath and Evans (1994), Johnston and Pattie (1991), Lanoue and Bowler (1992), Niemi et al. (1992)). Strategic voting has also been documented in elections in Canada (Blais and Nadeau (1996), Germany (Spenkuch (2013))\footnote{The author’s finding is particularly significant as he shows that 35% of voters abandon their most preferred candidate if they consider he has no possibility of winning the race.} and the US (Abramson et al. (1992)).

Answering the questions of when and to what extent strategic voting occurs is one of the central motivations behind the theory of voting. Seminal works by Palfrey (1989), Cox(1994,1997) and Myerson and Webber (1993) identify conditions that induce strategic voting. In those models, the incentive to vote strategically (or the lack thereof) is manifested in a rather stark way: in equilibrium, voters either abandon all but two candidates or split their votes evenly.\footnote{The literature typically refers to the former as a Duvergerian Equilibrium and the latter as a Non-Duvergerian Equilibrium. Fey (1997) provides a rationale based on dynamic} More recent works by Picketty (2000) and Myatt (2007)
provide a theoretical foundation for partial strategic voting by exploring the
effect of incomplete information on the incentive for strategic voting.\(^4\)

Existing works on strategic voting belong to the rational voting paradigm,
where analysis is often complicated by issues such as multiplicity of equilibria.
In this paper, we explore strategic voting in an ethical voting model in the
spirit of Feddersen and Sandroni (2006a, 2006b). We assume that a fraction
of voters are ethical in that, instead of voting to maximize private utility,
they follow a voting rule that maximizes the aggregate welfare of those with
similar political ideology.\(^5\)

The proposed model is tractable and delivers comparative statics of strate-
gic voting with respect to various electoral parameters such as the importance
of the election and the degree of partisanship. In particular, the model recon-
ciles the seemingly contradictory claims in the literature regarding the rela-
tionship between strategic voting and the popularity of the Condorcet loser.
Cain (1978) provides an informal argument that strategic voting disappears
when the popularity of the Condorcet loser is high. On the other hand,
Myatt (2007) argues that strategic voting is increasing in the popularity of
the Condorcet loser, although he claims the result “runs against established
intuition” (Myatt pg. 269). Empirical analyses on this issue have also failed
to yield an unified conclusion.\(^6\) In our model, strategic voting increases in
the popularity of the Condorcet loser in some range of the parameter that
captures popularity and decreases for others. We provide a stylized account
of UK elections in the 1980s to support this result.

\(^4\)Empirical studies have shown that various degrees of strategic voting have occurred in
UK and Canadian elections (see Cain (1978), Niemi, Whitten and Franklin (1992), Heath
and Evans (1994), Blais and Nadeau (1996)).

\(^5\)The ethical voters in our model constitute an example of rule-utilitarians (see
Harsanyi 1980 and 1977 for a discussion)

\(^6\)Fisher (2000) concludes a positive relationship while Blais and Nadeau (1996) show
the opposite correlation for the 1988 Canadian Election.
In addition to providing a more comprehensive understanding of strategic voting, our model contributes to the emerging literature on ethical voters/agents. While the application of ethical agents to economic modeling is relatively new, the philosophical roots of the concept have been well established (see Harsanyi (1977) and (1980) for a background). Harsanyi was the first to discuss, via examples, the implications of ethical agents in an electoral setting. Feddersen and Sandroni (2006a) build on Harsayni’s idea to model turnout in large elections, and their results are shown to fit the data better than rational voting models (see Coate and Conlin (2004)). Note that Feddersen and Sandroni (2006a) study two-candidate elections and therefore cannot account for strategic voting. Recently, Piolatto and Schuett (2012) use ethical voters to explain the demand for political news. The results in these studies, as well as the ones in this paper, are a testament that the ethical agent framework is particularly useful not only because of their tractability but, more important, because they can deliver novel insights. Thus, ethical voters should not be seen as a mere technical trick to simplify analysis of mass behavior. Evidence suggests that ethical motives, including a desire to follow rules that the agent considers as morally superior, play a significant role in mass behavior. It should not be surprising that ethical agent models can provide a reasonable characterization of voting behavior and other phenomena such as political activism and charity, where motives that transcend self-interest are likely to be present.

The rest of this paper is organized as follows; the model is described in the next section; in Section 3, we characterize the ethical voting rule and present our comparative statics results; Section 4 provides several extensions to the general model, including one which allows us to analyze the turnout.

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7In Feddersen and Sandroni (2006a and 2006b), ethical voters have opposing objectives and act strategically.
8Turnout can be studied jointly with strategic voting in our model, see section 4.
9For example, Blais (2000) shows that many voter vote out of a sense of civic duty.
and strategic voting decisions jointly; Section 5 concludes.

2 The Model

We study large elections with three candidates, one of whom is a Condorcet loser. For convenience, we shall refer to the Condorcet loser as the extremist and the two other candidates as centrists: the center-left and the center-right. To approximate a large election, we assume a measure one of voters. Voting is costless and mandatory (this is relaxed in Section 4) and the winner is decided by majority. It is commonly known to all players that a measure $k_e$ of the voters will vote for the extremist. Thus $k_e$ represents the popularity of the extremist. The assumption that the extremist is the Condorcet loser implies that $k_e < \frac{1}{2}$. The rest of the electorate are centrist supporters.

We assume that centrist supporters obtain utility $w > 0$ in the event that either centrists wins. Alternatively, one can think of $-w$ as the disutility associated with the extremist winning. Thus, $w$ measures how much stake the centrist voters have in the election, and we interpret $w$ as the importance of the election for the centrist supporters. In general, different centrist voters may attach different values to a center-left victory and a center-right victory, but this generalization will not alter our results in any qualitative way (see Section 4).

The centrist voters are heterogeneous in their preferences over the centrist candidates. They are endowed with ideal points drawn independently from an uniform distribution on the interval $[0, 1]$, where 0 and 1 are the position of the center-left and center-right candidate respectively. Voter $x$ (we refer to a voter by his ideal point) incurs a cost of $\theta x$ for voting for the center-left
and a cost $\theta(1 - x)$ for voting for the center-right.\footnote{That is, we assume that the voters dislike expressing their support for a political platform that is not their ideal. Brennan (2001) states that in the context of a large election, “the most important question in determining who (a voter) votes for is what she is prepared to cheer for (and boo) in the electoral context.” (quote from Brennan pg. 226). Studies by Fischer (1996), Kan and Yang (2001) and Laband et al. (2009), among others, provide evidence that expressive considerations are significant factors in voting.} We interpret $\theta$ as the degree of partisanship among centrist voters, since higher $\theta$ means greater utility loss for $x$ if he votes for his less preferred centrist. In sum, voter $x$’s (personal) utility of voting for the center-left and the center-right are respectively:

\[
wp - \theta x \\
wp - \theta(1 - x)
\]

where $p$ is the probability that a centrist candidate is elected. We assume that some voters vote according to their personal utility, and thus they will vote for the centrist closer to their ideal points. The rest of the centrist voters are ethical, whose characteristic is described below. We assume that the fraction of ethical voters, $q_c$, is unknown but all players have prior belief that it is distributed uniformly on $[0, 1]$.

The ethical voters follow a voting rule that maximizes the expected welfare of the centrist supporters (See Harsayni(1977,1980) for justification). We refer to this rule as the ethical voting rule. A given ethical voting rule may require some ethical voters to vote against their personal utility.\footnote{One can formalize the notion of ethical voters by assuming that they obtain utility $\phi > \theta$ for following the ethical voting rule. See Feddersen and Sandroni (2006a) for an example.} Without the loss of generality, the voting rule is of the cut-off type: a voting rule $\sigma_c \in [0, 1]$ instructs the ethical voters with ideal points $x > \sigma_c$ to vote for the center-right, and those with ideal points $x \leq \sigma_c$ to vote for center-left. Fix a
particular rule $\sigma_c$, there is an expected social cost for centrist voters (ethical and non-ethical), which is:

$$E\left[q_c\theta(\sigma_c^2 - \sigma_c + \frac{1}{2}) + (1 - q_c)\theta\frac{1}{4}\right] = \frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) + \frac{\theta}{8}$$

Let $p(\sigma_c)$ denote the probability of a centrist victory induced by the voting rule $\sigma_c$, the aggregate welfare of centrist supporters can be written as:

$$F(\sigma_c) = wp(\sigma_c) - E\left[q_c(\theta(\sigma_c^2 - \sigma_c + \frac{1}{2})) + (1 - q_c)\theta\frac{1}{4}\right]$$

$$= wp(\sigma_c) - \frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) - \frac{\theta}{8}$$

Let $\sigma_c^*$ denote the maximizer of $F(\sigma_c)$. It exists because $F(\sigma_c)$ is a continuous function on a compact domain. It is easy to see that $F(\sigma_c)$ is symmetric around $\frac{1}{2}$ because the ideal points are drawn from an uniform distribution. The symmetry of the objective function with regard to $\sigma_c$ implies that the rule $\sigma_c$ is equivalent to the rule $1 - \sigma_c$. Thus, we restrict the domain of the maximization problem to $[0, \frac{1}{2}]$. That is, we assume without loss of generality that ethical voters rally behind the center-right.\(^\text{12}\) Observe that the ethical voters engage in strategic voting if $\sigma_c^* \in [0, \frac{1}{2}]$, since ethical voters with ideal points in $[\sigma_c^*, \frac{1}{2}]$ are voting against their personal utility. We interpret $2(\frac{1}{2} - \sigma_c^*)$ as the degree of strategic voting.

For a given $\sigma_c$, the probability of centrists winning is the probability that

\(^{12}\)Skewness in the distribution of ideal points is sufficient to break the indifference between the two rules and dispose of this assumption.
the center-right receives more votes than the votes for the extremist, $k_e$:

$$
\Pr \left( (1 - k_e) \left[ (1 - q_c) \frac{1}{2} + q_c(1 - \sigma_c) \right] \geq k_e \right)

= \Pr \left( q_c \geq \frac{k_e}{\frac{1-k_e}{2} - \sigma_c} \right)

$$

Since $q_c$ is distributed uniformly, this means:

$$
p(\sigma_c) = \max \left\{ 0, 1 - \frac{\frac{k_e}{1-k_e} - \frac{1}{2}}{\frac{1}{2} - \sigma_c} \right\}
$$

and the objective function $F(\sigma_c)$ can be written as:

$$
F(\sigma_c) = w \cdot \max \left\{ 0, 1 - \frac{\frac{k_e}{1-k_e} - \frac{1}{2}}{\frac{1}{2} - \sigma_c} \right\} - \frac{\theta}{2} (\sigma_c^2 - \sigma_c + 1) - \frac{\theta}{8}
$$

### 3 Results

For $k_e \leq \frac{1}{3}$, the centrists win with certainty. 13 Thus, maximizing $F$ is equivalent to minimizing the expected social cost. It follows that in this case the ethical voters will split their vote (i.e. $\sigma_c^* = \frac{1}{2}$). This is intuitive: there is no incentive for strategic voting if it has no bearing on the outcomes. For the remainder of the paper, we focus on the non-trivial scenario $\frac{1}{3} < k_e < \frac{1}{2}$.

To simplify notations, denote $\tilde{k}_e = \frac{k_e}{1-k_e}$ (i.e. the ratio of extremist supporters to centrist supporters). Note that $\tilde{k}_e$ is a monotonic transformation of $k_e$ and thus they have the same interpretation (i.e. the popularity of the

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13We assume here the centrist candidate wins the election in the event of a tie.
extremist). Moreover, the restriction that \( \frac{1}{3} < k_e < \frac{1}{2} \) translates to the restriction that \( \frac{1}{2} < \tilde{k}_e < 1 \).

**Proposition 1.** There exists a threshold value of \( \tilde{k}_e \) (call it \( \bar{k}_e \)) such that the ethical voters split their votes (i.e. \( \sigma^*_c = \frac{1}{2} \)) if \( \tilde{k}_e \geq \bar{k}_e \), and vote strategically (i.e. \( \sigma^*_c < \frac{1}{2} \)) if \( \tilde{k}_e < \bar{k}_e \). Moreover, whenever strategic voting is optimal, \( \sigma^*_c < 1 - \bar{k}_e \).

**Corollary 1.** \( \bar{k}_e \) is increasing in \( w \) and decreasing in \( \theta \).

Proposition 1 agrees with claims by Cain (1978) and others that the incentive to vote strategically disappears when the Condorcet loser has high popularity (though not high enough for a majority). Recall our interpretation of \( w \) as the importance of election and \( \theta \) as the degree of partisanship of centrist voters. Corollary 1 therefore offers two empirically testable implications: one is more likely to observe strategic voting in important elections (i.e. presidential and parliamentary elections) and when the centrist supporters are less partisan in their preferences of the two centrists. Proposition 2 below provides some characterization of the threshold value \( \bar{k}_e \).

**Proposition 2.**

1. For \( \frac{\theta}{w} \leq \frac{8}{3} \), \( \bar{k}_e = 1 - \frac{\theta}{16w} \).
2. For \( \frac{8}{3} < \frac{\theta}{w} \), \( 1 - \frac{\theta}{16w} < \bar{k}_e < \min\{\frac{1}{2} + \frac{\theta}{8w}, \frac{1}{2} + \left(\frac{\theta}{w}\right)^{-\frac{1}{2}}\} \)

**Corollary 2.** Complete strategic voting (i.e. \( \sigma^*_c = 0 \)) occurs if and only if \( \frac{1}{2} + \frac{\theta}{8w} \leq \bar{k}_e \leq 1 - \frac{\theta}{16w} \).

Figure 1 below summarizes our results so far. We have explored conditions under which strategic voting is optimal. This is only one aspect of the comparative statics. We will now relate the degree of strategic voting and the electoral parameters conditional on strategic voting being optimal.
**Proposition 3.** Given strategic voting is optimal (i.e. $\sigma^* < \frac{1}{2}$), the degree of strategic voting is increasing in the importance of election and the popularity of the extremist (i.e. $\frac{\partial \sigma^*}{\partial w} \leq 0$ and $\frac{\partial \sigma^*}{\partial k_e} \leq 0$), while it is decreasing in the cost of voting (i.e. $\frac{\partial \sigma^*}{\partial \theta} \geq 0$). The inequalities are strict if $\sigma^* > 0$.

The comparative statics with respect to the importance of elections, $w$ and the degree of partisanship among centrists supporters, $\theta$, are expected given their interpretations. The more interesting observation is that for moderate popularity of the extremist (i.e. values of $k_e$ that induce strategic voting), small increases in the popularity induce greater incentive to vote strategically. Combine this observation with Proposition 1, we see a non-monotonic (and discontinuous) relationship between strategic voting and the popularity of the extremist (see Figure 2). This new insight reconciles the contradictory empirical findings on this issue (see Cain (1978), Fisher (2000) and Blais and Nadeau (1996)): if the support for the extremist is low and variation is small in the data, then the econometrician is likely to observe a positive correlation between the popularity of the extremist and strate-
gic voting; if the extremist support is high and variations large, then the econometrician is likely to observe a negative correlation.

\[ 2 \left( \frac{1}{2} - \sigma^*_c \right) \]

Figure 2: Degree of Strategic Voting as a Function of Extremist Support

Note that Proposition 3 implies that the probability of a centrist victory, \( p(\sigma^*_c) \), is decreasing in \( \theta \) and increasing in \( w \). The effect of an increase in extremist popularity on the probability of a centrist victory is not obvious, since strategic voting may increase in response to an increase in extremist popularity. The following result show that adjustment in strategic voting has only second order effect on the probability of a centrist victory.

**Proposition 4.** The probability of centrists winning is decreasing in the popularity of the extremist.

### 3.1 UK Elections in the Thatcher Years

We now present stylize facts on the UK elections during the last Conservative Era (1979 - 1997), which provide good anecdotal evidence in support of our
main observations per Proposition 1 and 3. In particular, the popularity of the Conservative Party had the predicted effect on the split of votes between the Labour and Liberal Democrats (LD), the two main oppositions to the Conservatives.

The Conservative Party and in particular its leader, Margaret Thatcher, dominated UK politics in the 1980s. In 1979, the Conservative Party defeated the incumbent Labor Party in the general election, although at that time Margaret Thatcher was not the popular figure she later became. Four years later, the Conservatives won the reelection on a wave of popular support, thanks to the victory in the Falklands War as well as improved economic conditions. After another victory in 1987, Thatcher resigned in 1990 and was succeeded by John Major, who did not have the charisma nor the popularity of Thatcher.

Our model suggests that in 1979, voters who opposed the Conservatives would have rallied behind either the Labour or the Liberal Democrats because of the moderate popularity of the Conservatives. And those voters would split the vote in 1983 due to the high popularity of the Conservatives. Noticing the weakness of the Conservatives in 1992, the ethical voters would have acted strategically in that election but only to a limited extent.

The trend of the outcomes in these elections supports our conclusions.\textsuperscript{14} In 1979, the Labour/LD split of the center-left votes was 73 to 27 percent, which indicates a high degree of strategic voting. In 1983, it was an almost even split, 52 to 48 percent. In 1992, the split was 66 to 34 percent, which suggests some degree of strategic voting, but not as high as the one registered in 1979. Empirical studies on these elections also lead to the same conclusions. For example, Heath and Evans (1994) estimate that strategic voting

\textsuperscript{14}In the UK, each Member of Parliament represents a separate constituency. However, the panorama of the national vote (in particular, the swing in the vote) is widely used in the UK as an indicator of the swing in embattled constituencies.
accounted for 9% of the vote in the 1992 election, while Johnston and Pattie (1991) find that it only accounted for 5.1% of all votes in 1983.

4 Extensions

4.1 Heterogeneity in $w$

So far, the centrist voters obtain $w$ regardless of which centrists wins. In general, a centrist voter may feel differently about the center-left candidate winning and the center-right candidate winning. Let $w_l : [0, 1] \rightarrow \mathbb{R}_{++}$ and $w_r : [0, 1] \rightarrow \mathbb{R}_{++}$ be two functions, where $w_l(x)$ and $w_r(x)$ are the utilities that voter $x$ receives when the center-left and the center-right candidate wins, respectively. Observe that, since there is a continuum of voters, the outcome cannot be affected by the decision of a single voter. Thus, partisan voters still vote for the candidate closer to their ideal point. The ethical voting rule, on the other hand, depends on the distribution of $w_l(x)$ and $w_r(x)$. In particular, the objective function $F(\sigma_c)$ can be expressed as:

$$p(\sigma_c)E(w_r) - \frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) - \frac{\theta}{8} \quad \forall \sigma_c \leq \frac{1}{2}$$

$$p(\sigma_c)E(w_l) - \frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) - \frac{\theta}{8} \quad \text{otherwise}$$

$F$ is not symmetric around $\frac{1}{2}$, unless $E(w_r) = E(w_l)$. More specifically, the ethical voting rule $\sigma_c^*$ belongs to $[0, \frac{1}{2}]$ if $E(w_r) > E(w_l)$ and $[\frac{1}{2}, 1]$ if $E(w_r) < E(w_l)$. Ethical voters simply rally behind the candidate whose victory provides the highest aggregate utility for the centrist voters. Hence, unlike our benchmark model, ethical voters will no longer be indifferent between supporting the center-right and the center-left.
It is straightforward to see that how one goes about solving for the optimal ethical rule or analyzing the comparative statics with regard to $\theta$ and $\tilde{k}_e$ will not be affected by this generalization. Comparative statics with respect to $E(w_r)$ and $E(w_l)$ are straightforward, keep in mind that one should now interpret $\max\{E(w_r), E(w_l)\}$ as the the importance of the election.

4.2 Turnout Decision

Suppose now the centrist voters also have to face a cost of turnout. In particular, suppose that the cost for a voter of going to the polls is $\theta t$, where $t$ is a uniform random variable over $[0, 1]$ independent of $x$. We assume the scaling factor $\theta$ is the same for $x$ and $t$ for simplicity. In sum, the voters are now uniformly distributed in the set: $[0, 1]^2$, where one dimension represents the expressive cost and the other the turnout cost.

Given the modification described above, it is easy to check that the ethical voting rule will be a constant $0 \leq \sigma_c \leq 2$ such that voters for whom $(1 - x) + t \leq \sigma_c$ will vote for the center-right candidate, and abstain if $(1 - x) + t > \sigma_c$. This means that, contrary to the case with no turnout cost, it is possible that some voters with a strong preference for the center-right candidate will not vote while those who prefer the center-left will do so. Hence, the model with turnout costs implies that it is possible to observe, in the same election, a certain degree of strategic voting and significant abstention from the supporters of the party that is receiving those strategic votes.

While we will not present the complete characterization of the problem with turnout costs, it is still convenient to portray the basic characteristics

\footnote{Simply take $\max\{E(w_l), E(w_r)\}$ as $w$ and when $E(w_l) > E(w_r)$, the statement of the result should be modified to reflect the fact that $\sigma^* \in [\frac{1}{2}, 1]$ is the optimal solution.}
of the problem that ethical voters face. These will show the reader that the nature and structure of the problem do not change. As before, ethical voters select a rule $\sigma_c$ that maximizes the expected social welfare of their group. Now, for a given $\sigma_c$, the expressions for the probability that a centrist candidate wins, $p(\sigma_c)$, and the cost of voting for the centrist group, $C(\sigma_c)$, are as follows:

$$p(\sigma_c) = \begin{cases} \max(1 - \frac{2k_e}{\sigma_c}, 0) & \text{if } 0 \leq \sigma_c \leq 1 \\ 1 - \frac{2k_e}{2-(2-\sigma_c)^2} & \text{if } 1 \leq \sigma_c \leq 2 \end{cases} \tag{1}$$

$$C(\sigma_c) = \begin{cases} \theta \sigma_c^3 & \text{if } 0 \leq \sigma_c \leq 1 \\ \theta \left(\frac{\sigma_c(\sigma_c+3)}{2} - \frac{\sigma_c^3+4}{3}\right) & \text{if } 1 \leq \sigma_c \leq 2 \end{cases} \tag{2}$$

The main difference between this setup and our benchmark model is the change in the slopes of $p(\sigma_c)$ and $C(\sigma_c)$ at $\sigma_c = 1$. However, the expressions that correspond to when $\sigma_c$ is between 0 and 1 are not relevant as the optimal $\sigma_c$ will either be 0 or be in the interval $[1, 2]$. When $\sigma_c \in [0, 1]$, at most half of the centrists supporters are voting. Thus, given the restrictions on $k_e$, centrists would be defeated with certainty. This means that ethical voters would be better off by abstaining completely. Hence, the range of the solution is similar to the one observed in the problem with no turnout costs, in which the optimal ethical voting rule lies in $[0, 1 - \tilde{k}_e) \cup \{\frac{1}{2}\}$. Moreover, the fact that $\sigma^*_c$ cannot be in the interval $[0, 1]$ implies that if turnout from ethical voters increases, the fraction of agents engaging in strategic voting will also increase.
5 Conclusions

In this paper, we construct an ethical voter model to study strategic voting. The model is parsimonious and provides a clear mechanism to explain when and to what degree strategic voting occurs. Our results shed light on puzzling observations. In particular, we are able to show how an increase in the popularity of the extremist candidate can lead to either an increase or a decrease in the degree of strategic voting, depending on the values of the electoral parameters.

The tractability of the ethical voter framework means that potentially interesting modifications can be carried out. For example, a model with an arbitrary number of candidates may be worth examining. While established democracies like the Canada and the UK have three main parties, this is not the case for several developing democracies. It may also be fruitful to extend the model to a dynamic setting, where one can examine the incentives to form coalitions for parties that derive their support from the same subset of the electorate.

The present study is an example of how ethical voter models can constitute useful alternatives to rational agent-based theories. In general, ethical agents can be used to explain how individuals act in contexts where an individual’s action has negligible impact on the outcome. Therefore, ethical agent models may prove useful for the analysis of collective phenomena such as consumer boycotts, mass rallies and recycling. As is the case for strategic voting, it is possible that the application of ethical agent models to these contexts leads to new insights.
References


A Mathematical Appendix

Proposition 1

Proof. Since

\[ 1 - \tilde{k}_e - \frac{1}{2} \leq 0 \forall \sigma_c \in [1 - \tilde{k}_e, \frac{1}{2}], \]

the objective function becomes \( F(\sigma_c) = -\frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) - \frac{\theta}{8} \) within the interval \([1 - \tilde{k}_e, \frac{1}{2}]\), and it is maximized at \( \frac{1}{2} \). Therefore, one can reduce the choice domain of the maximization problem to \([0, 1 - \tilde{k}_e) \cup \{\frac{1}{2}\}\). The solution to the maximization problem can be found by first finding the optimal solution \( \tilde{\sigma}_c \) on the interval \([0, 1 - \tilde{k}_e]\). If \( \tilde{\sigma}_c \equiv 1 - \tilde{k}_e \), then the global optimum is \( \sigma^*_c = \frac{1}{2} \) since \( F(1 - \tilde{k}_e) < -\theta \cdot \frac{1}{4} = F(\frac{1}{2}) \). Otherwise, a comparison between \( F(\tilde{\sigma}_c) \) and \( F(\frac{1}{2}) \) determines the global optimum. It is easy to verify that the second order derivative of \( F \) within the interval \([0, 1 - \tilde{k}_e] \) is negative because \( \tilde{k}_e > \frac{1}{2} \) and \( \sigma_c < \frac{1}{2} \). Consequently, the first order condition is sufficient and necessary for \( \tilde{\sigma}_c \).

Let \( F^*(w, \theta, \tilde{k}_e) = \max_{\sigma_c \in [0,1-\tilde{k}_e]} F \). By Berge’s theorem, \( F^* \) is continuous in its arguments, in particular it is continuous in \( \tilde{k}_e \). We will show below the existence of a \( \tilde{k}_e \) for which \( F^* \) exceed \(-\frac{\theta}{4}\) and another \( \tilde{k}_e \) for which \( F^* \) is less than \(-\frac{\theta}{4}\). This will allow us to apply the intermediate value theorem and
conclude there exist \( \bar{k}_e \) such that \( F^*(w, \theta, \bar{k}_e) = -\frac{\theta}{4} = F(\sigma_e = \frac{1}{2}) \). And this \( \bar{k}_e \) is the threshold we desire because \( F^* \) is also monotonic in \( \bar{k}_e \).

It is straightforward to see that for \( \tilde{k}_e \) sufficiently low, \( F^*(w, \theta, \tilde{k}_e) > -\frac{\theta}{4} = F(\sigma_e = \frac{1}{2}) \). Next, we would like to show that the reverse inequality holds for sufficiently high \( \tilde{k}_e \). Note that the inequality \( F^*(w, \theta, \tilde{k}_e) < -\frac{\theta}{4} \) is necessary and sufficient condition for \( \frac{1}{2} \) being optimum, and a sufficient condition for the optimality of \( \frac{1}{2} \) is:

\[
-\frac{\theta}{4} \geq \max_{\sigma_e \in [0, 1 - \tilde{k}_e]} \{wp(\sigma_e)\} - \min_{\sigma_c \in [0, 1]} \left\{ \frac{\theta}{2} \left( \sigma_c^2 - \sigma_c + \frac{1}{2} \right) \right\} - \frac{\theta}{8}
\]

Because \( \frac{1}{2} = \arg\min_{\sigma_e \in [0, 1]} \{\frac{\theta}{2} \left( \sigma_e^2 - \sigma_e + \frac{1}{2} \right)\} \), \( \min_{\sigma_c \in [0, 1 - \tilde{k}_e]} \left\{ \frac{\theta}{2} \left( \sigma_c^2 - \sigma_c + \frac{1}{2} \right) \right\} = \frac{\theta}{8} + \epsilon \) for some positive \( \epsilon \). Additionally, \( \max_{\sigma_e \in [0, 1]} \{wp(\sigma_e)\} = w(2 - 2\tilde{k}_2) \).

Rewriting the sufficient condition, we have:

\[
-\frac{\theta}{4} \geq w(2 - 2\tilde{k}_2) - \frac{\theta}{4} - \epsilon \iff \tilde{k}_e \geq 1 - \frac{\epsilon}{2w}
\]

Thus, for \( \tilde{k}_e \) sufficiently high, \( \frac{1}{2} \) is the optimum and equivalently, \( F^*(w, \theta, \tilde{k}_e) < -\frac{\theta}{4} \).

\[ \Box \]

**Corollary 1**

**Proof.** Observe that \( F^* \) is increasing in \( w \) and decreasing in \( \theta \), and to satisfy \( F^*(w, \theta, \tilde{k}_e) = -\frac{\theta}{4} \), \( \tilde{k}_e \) must increase in \( w \) and decrease in \( \theta \). \( \Box \)

**Proposition 2**

**Proof.** Note first that, \( F(0) \geq F(\frac{1}{2}) \iff w \left( 1 - \frac{\tilde{k}_e - \frac{1}{2}}{2} \right) - \frac{\theta}{4} \geq -\frac{\theta}{8} \iff \)

21
1 - \frac{\theta}{16w} \geq \tilde{k}_e. This implies that 1 - \frac{\theta}{16w} \geq \tilde{k}_e is sufficient for strategic voting. Furthermore, if \frac{1}{2} + \frac{\theta}{8w} \leq \tilde{k}_e, then \frac{\partial F(0)}{\partial \sigma} = -\frac{w(\tilde{k}_e - \frac{1}{2})}{\frac{1}{2}} + \frac{\theta}{2} \leq 0. This means that for \frac{1}{2} + \frac{\theta}{8w} \leq \tilde{k}_e, \sigma^* < \frac{1}{2} \implies \sigma^* = 0. Thus, the first result follows because \frac{1}{2} + \frac{\theta}{8w} \leq \tilde{k}_e is a sufficient condition for vote splitting. However, this condition is vacuous for \frac{\theta}{w} \geq 4. We will now show that \tilde{k}_e > \frac{1}{2} + \sqrt{\frac{w}{\theta}} is a sufficient condition for vote splitting, and this will complete our proof.

Observe that on the interval [0, 1 - \tilde{k}_e], F(\sigma_c) = w \left[ 1 - \frac{\tilde{k}_e - \frac{1}{2}}{\frac{1}{2} - \sigma_c} - \frac{\theta}{2}(\sigma_c^2 - \sigma_c + \frac{1}{2}) - \frac{\theta}{8} \right] is a strictly concave function. It means that if \frac{\partial F}{\partial \sigma_c}(1 - \tilde{k}_e) \geq 0, \frac{1}{2} is the global optimum since \( F(\frac{1}{2}) > F(1 - \tilde{k}_e) \geq F(\sigma_c) \forall \sigma_c \in [0, 1 - \tilde{k}_e]. \) Now,

\[
\frac{dF}{d\sigma_c}(1 - \tilde{k}_e) = -\frac{w(\tilde{k}_e - \frac{1}{2})}{(\frac{1}{2} - \tilde{k}_e)^2} - \theta(1 - \tilde{k}_e) + \frac{\theta}{2} = -\frac{w}{\tilde{k}_e - \frac{1}{2}} - \theta(1 - \tilde{k}_e) + \frac{\theta}{2}
\]

and \(-\frac{w}{\tilde{k}_e - \frac{1}{2}} - \theta(1 - \tilde{k}_e) + \frac{\theta}{2} \geq 0 \iff \tilde{k}_e^2 - \tilde{k}_e + \frac{1}{4} - \frac{w}{\theta} \geq 0. \) Apply the quadratic formula, we see that for \( \tilde{k}_e \geq \frac{1 + \sqrt{\frac{4w}{\theta}}}{2} \) and \( \tilde{k}_e \leq \frac{1 - \sqrt{\frac{4w}{\theta}}}{2} \), the derivative of F is positive. However, note that it cannot be the case \( \tilde{k}_e \leq \frac{1 - \sqrt{\frac{4w}{\theta}}}{2} \) since \( \tilde{k}_e > \frac{1}{2} \) by assumption.

□

**Proposition 3**

**Proof.** We shall use the monotone comparative statics results from Milgrom & Shannon (1994): If the cross derivative of F with respect to the choice variable (i.e. \( \sigma_c \)) and the parameter of interest, and if the cross partial
is positive (negative), then the solution is increasing (decreasing) in that parameter. Observe that for \( \sigma^*_c \in [0, 1 - \tilde{k}_e) \), \( p(\sigma_c) = 1 - \frac{\tilde{k}_e - \frac{1}{2}}{2 - \sigma_c} \), it follows that \( \frac{\partial^2 F}{\partial w \partial \sigma_c} < 0 \), \( \frac{\partial^2 F}{\partial \theta \partial \sigma_c} > 0 \) and \( \frac{\partial^2 F}{\partial \tilde{k}_e \partial \sigma_c} < 0 \).

**Proposition 4**

*Proof.* When \( 0 < \sigma^*_c < \frac{1}{2} \), the first order conditions characterizes the optimum and the following expression is obtained: \( \sigma^*_c = \frac{1}{2} - \sqrt{\frac{w}{\theta}} \left( \tilde{k}_e - \frac{1}{2} \right) \).

Thus, we see that \( \frac{\partial \sigma^*_c}{\partial \tilde{k}_e} = -\frac{w}{\theta} \left( \frac{w}{\theta} (\tilde{k}_e - \frac{1}{2}) \right)^{-\frac{3}{2}} \). From the expression for the probability of the center winning the election, we obtain:

\[
\frac{\partial p(\sigma^*_c)}{\partial \tilde{k}_e} = -\frac{1}{2} - \sigma^*_c + (\tilde{k}_e - \frac{1}{2}) \frac{\partial \sigma^*_c}{\partial \tilde{k}_e} \frac{(\frac{1}{2} - \sigma^*_c)^2}{(\frac{1}{2} - \sigma^*_c)^2}
\]

Replacing our previous values for \( \sigma^*_c \) and \( \frac{\partial \sigma^*_c}{\partial \tilde{k}_e} \), we obtain that the numerator is equal to \( \frac{2}{3} \sqrt{\frac{w}{\theta}} \left( \tilde{k}_e - \frac{1}{2} \right) > 0 \). Since the denominator is also positive and the fraction is multiplied by -1, we get the probability of winning is decreasing when \( \sigma^* \) is interior. The results extends easily to the case where \( \sigma^* = 0 \) (refer to Proposition 1 and 2)

\[\Box\]